INCOME

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Why study Economic Growth?

- **Economic growth** is the phenomenon related to growth of income, living standards, etc.

- Since it is related to the overall increase of production, it is considered to be a "Macroeconomic phenomenon"
  - => basic unit of measurement is the national Outcome (i.e., Gross Domestic Product)

- However, keep in mind that economic growth is related to many structural factors acting within the economy (i.e., urbanization, education, innovation...) and, first of all, affecting firms' decisions:
  => ..."microeconomic" perspective
  => performance, productivity, intl competitiveness, etc…

Measuring «Output»

- **GDP (Gross Domestic Output)**: the value (at market prices) of all final goods and services produced in a country in a given year

The reason for including only final goods and not intermediate goods is clear:

- If a baker sells bread at 10 Euros, and it buys the flour from the miller by 5 Euros, and water electricity etc. are bought at 1 Euro, then adding them together will amount to 16 Euro.
- But if the baker would produce all by himself, it will produce only 10 Euro. We want the measure to be independent of the organization of production.

Measuring «Output»

Alternative ways to think about GDP:

- Sum of value added of all firms.
    - Value added is revenues minus intermediate goods (intermediate goods are revenues to their sellers => adding value added across all firms leaves only the final goods).
- Sum of workers' and firm owners' (K and L) income plus depreciation
- Another method is $Y \equiv C + G + I + EX - IM$

Measuring «Output»

Gross national income (GNI):

- total domestic and foreign output claimed by residents of a country
- Given by GDP + factor incomes earned by foreign residents - income earned in the domestic economy by nonresidents

Comparing «Output»

- Over time: control for inflation

  => “deflated” GDP (or “real” GDP)

- Across countries: convert to common currency (e.g. USD)
  - nominal exchange rate => may not be a good idea
  - control for differences in purchasing power

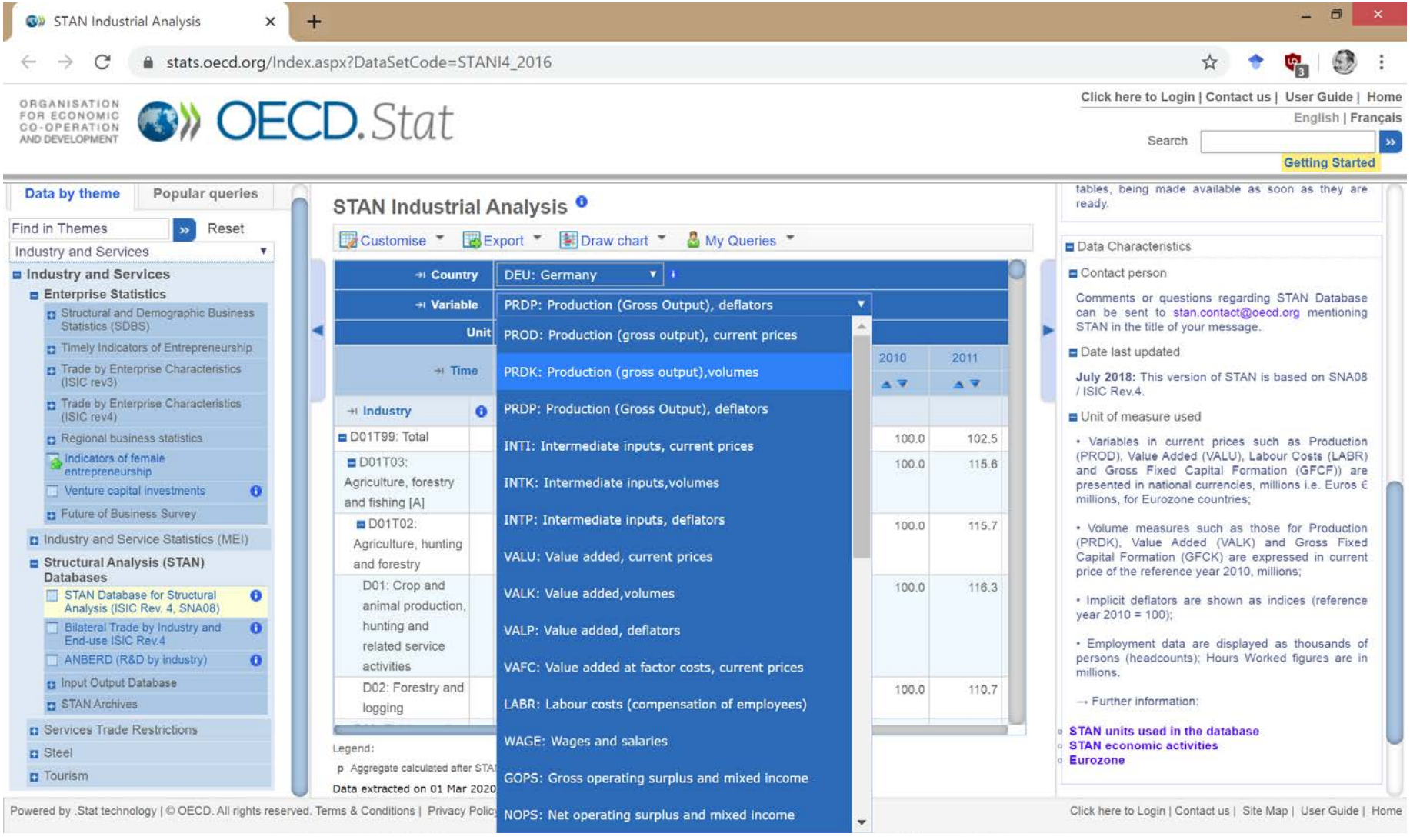
    => PPP (Purchasing Power Parity) exchange rates

Comparing «Output» over time: real GDP

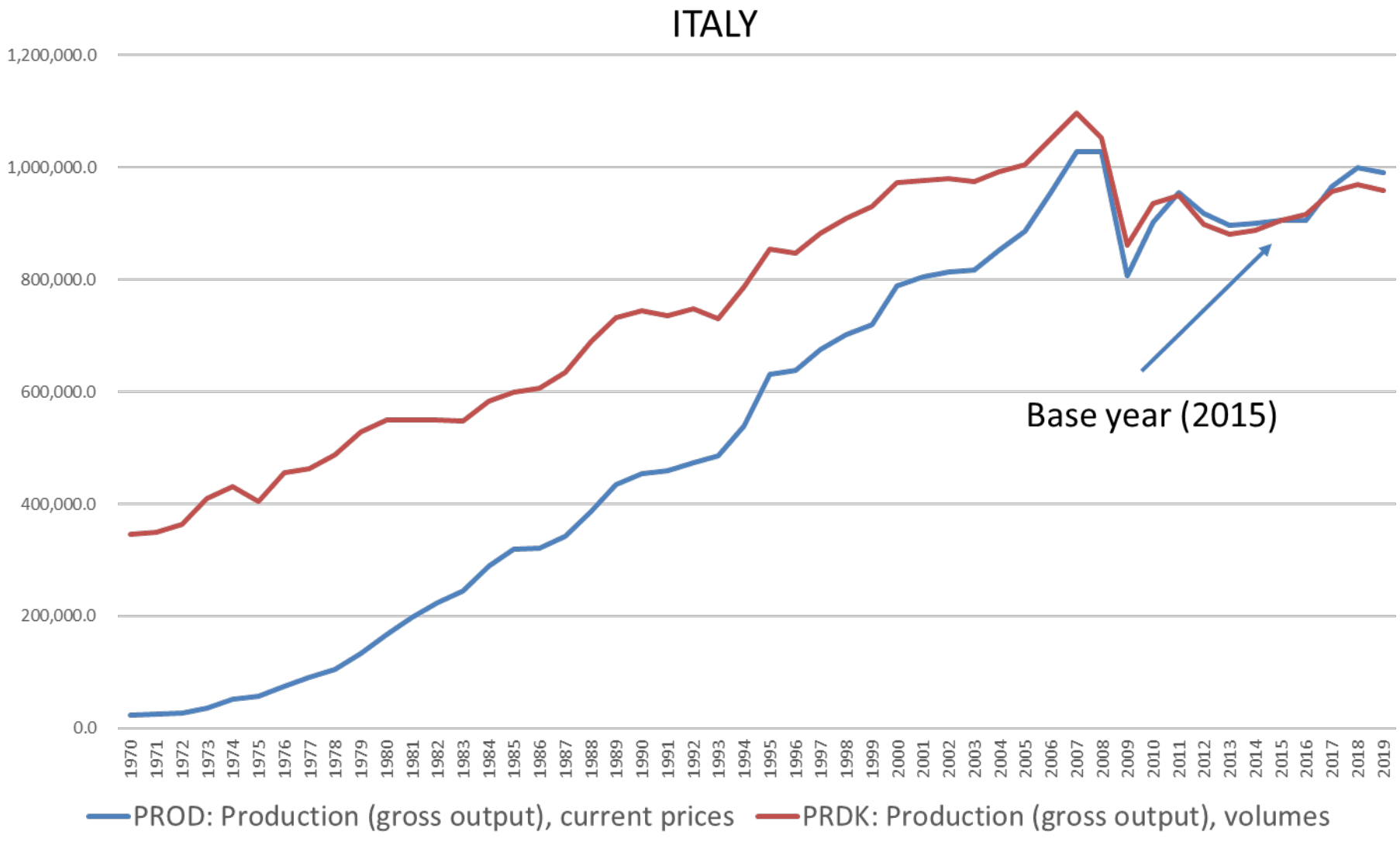
- By definition output is measured with prices, in money terms.
- To avoid the effect of changes in prices:
  - Real GDP (GDP at constant prices): a measure of real output)
    - => *Constant* prices (representative year) instead of *current* prices
    - => How? divide nominal output by price level to get the real quantity.

      Intuition: (nominal GDP) / (price index) = PY/P
      - price index:
        - » Avg price of *consumed* goods: CPI (Consumer Price Index)
        - » Avg price of *produced* goods: GDP deflator

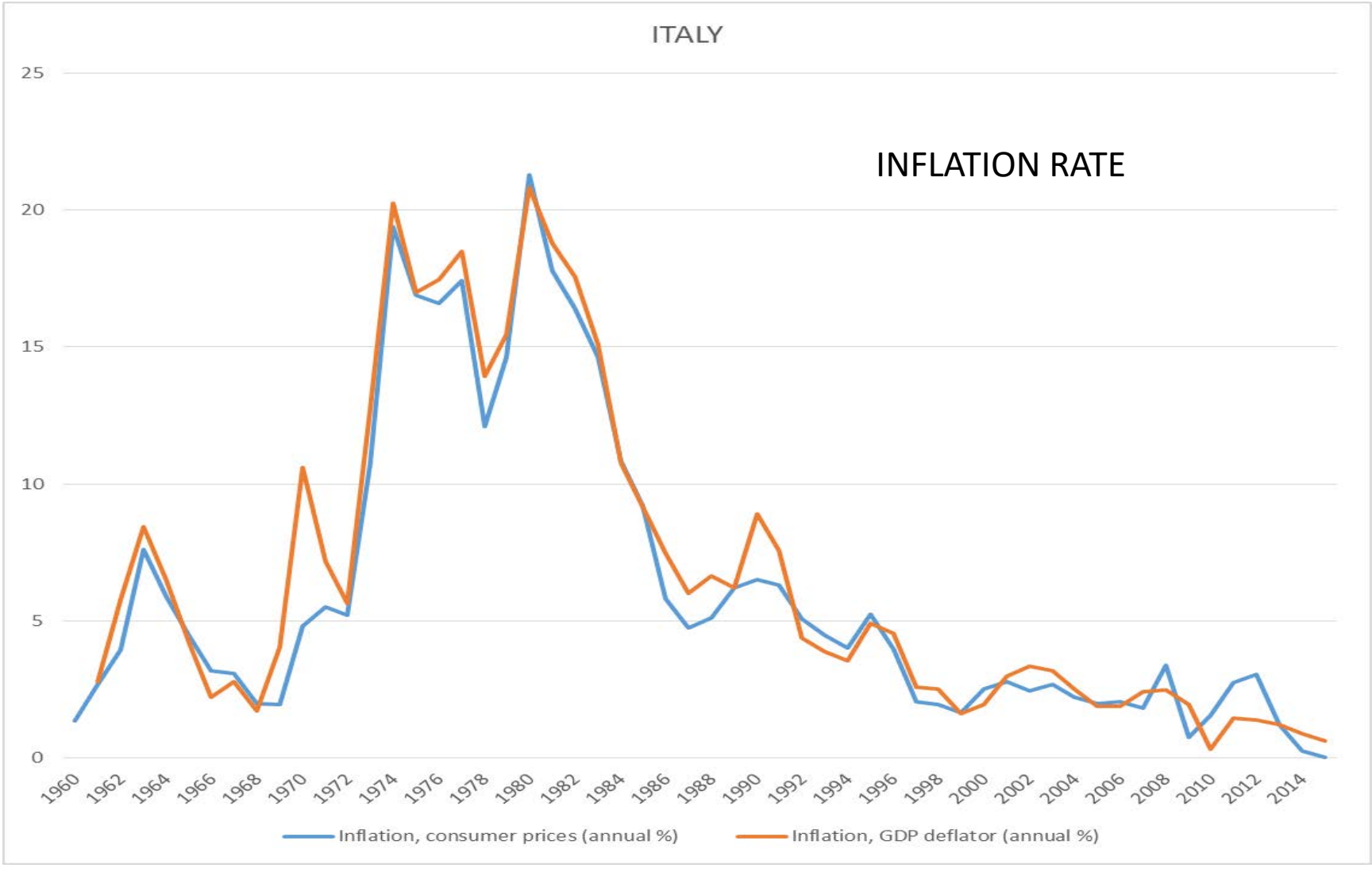
Comparing «Output» over time: real GDP …an example



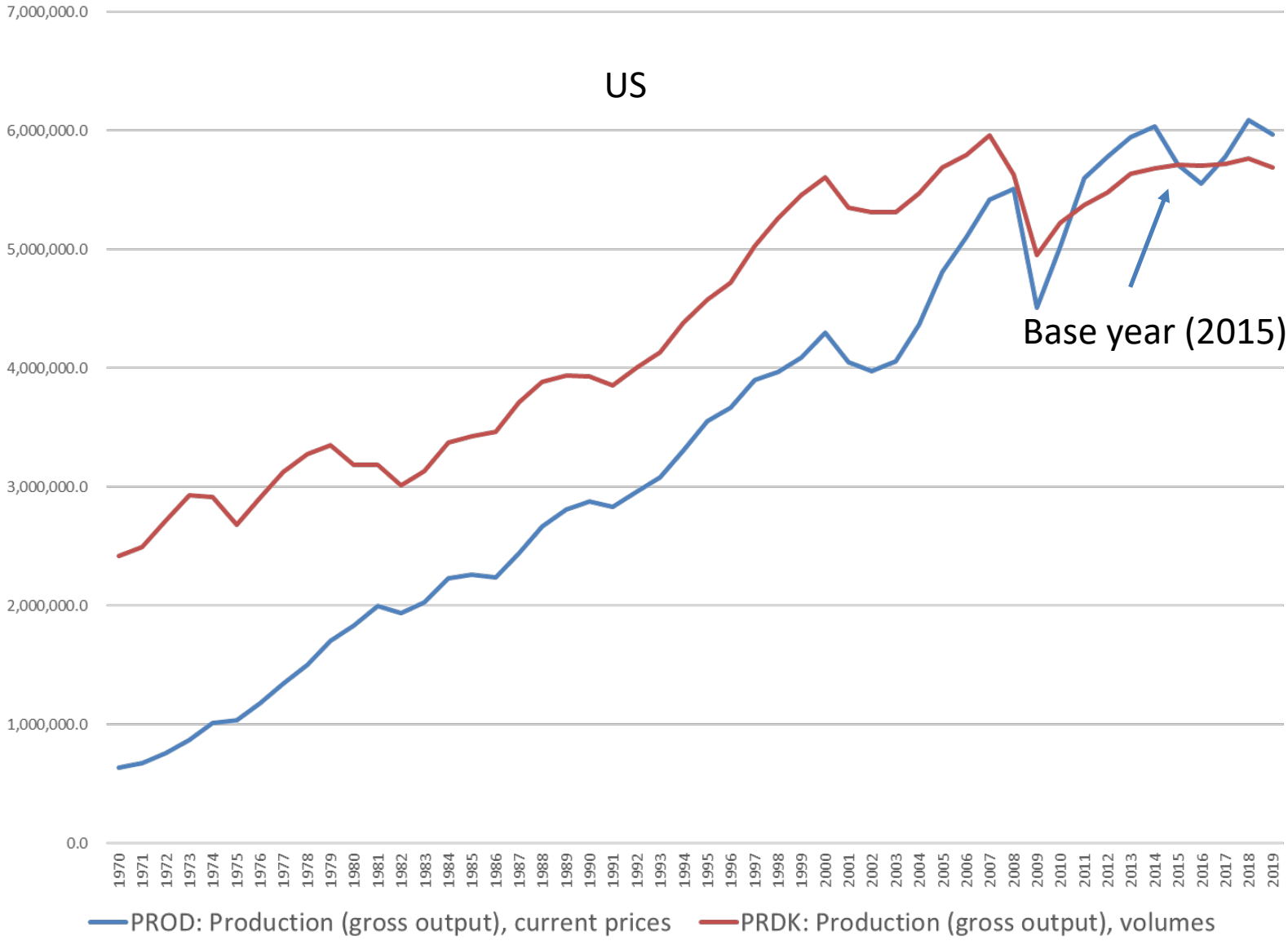
Comparing «Output» over time: real VS nominal GDP



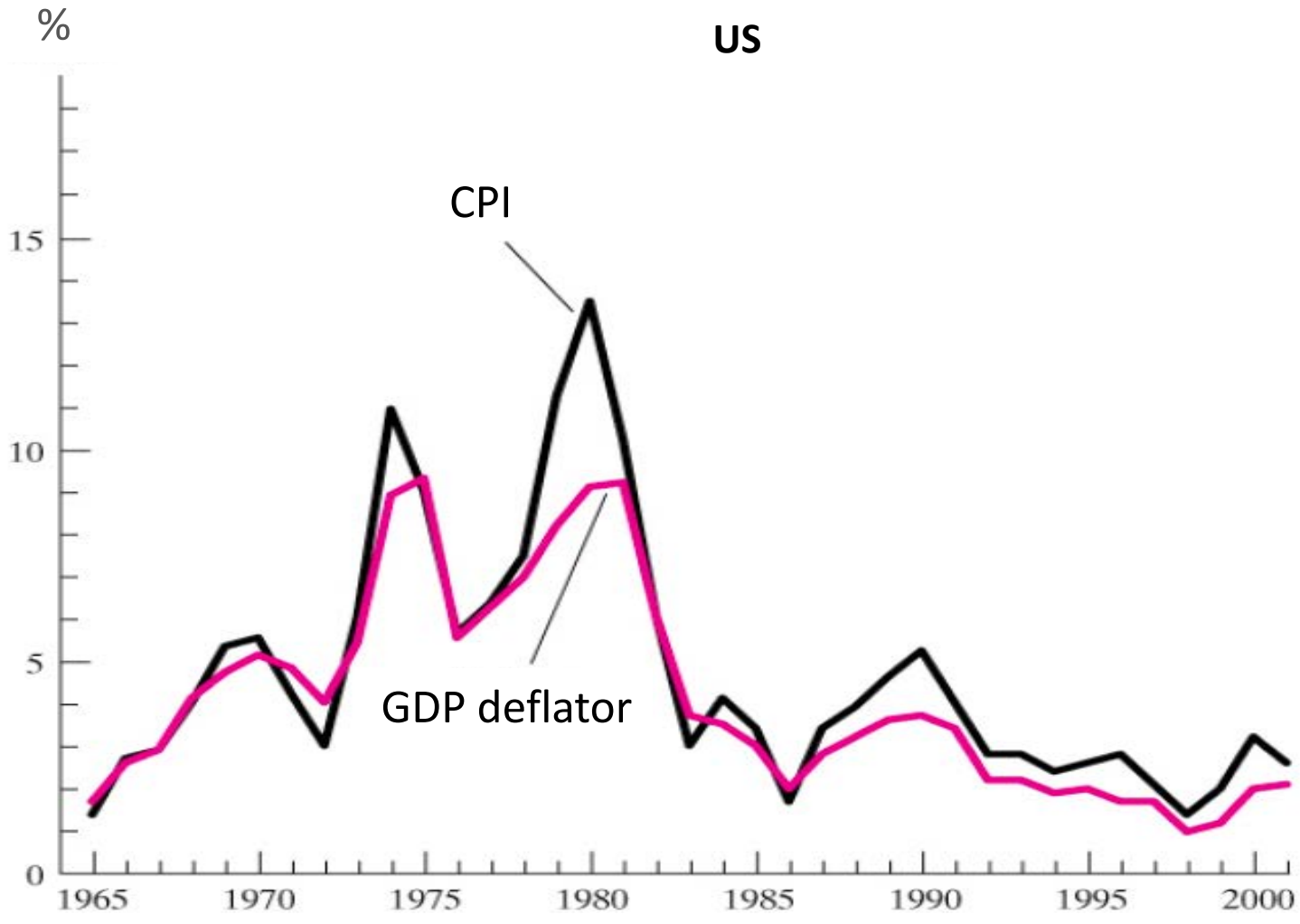
Comparing «Output» over time: real VS nominal GDP



Comparing «Output» over time: real VS nominal GDP



Comparing «Output» over time: «price index»



Comparing «Output» across countries: PPP (Purchasing Power Parity)

If we want to compare output across countries we need real variables that take in consideration different cost of living. Such data are called “PPP adjusted”.

Country	Production of Televisions per Capita	Production of Haircuts per Capita	Price of Televisions in Local Currency	Price of Haircuts in Local Currency	GDP per Capita in Local Currency
Richland	4	40	10	2	120
Poorland	1	10	10	1	20

PPP exchange rates calculated by measuring the cost of a basket of goods and services.

- E.g. basket = (1 TV, 10 haircuts).

Price of basket is $1\times10+10\times2=\mathbf{30}$ in R, $1\times10+10\times1=\mathbf{20}$ in P.

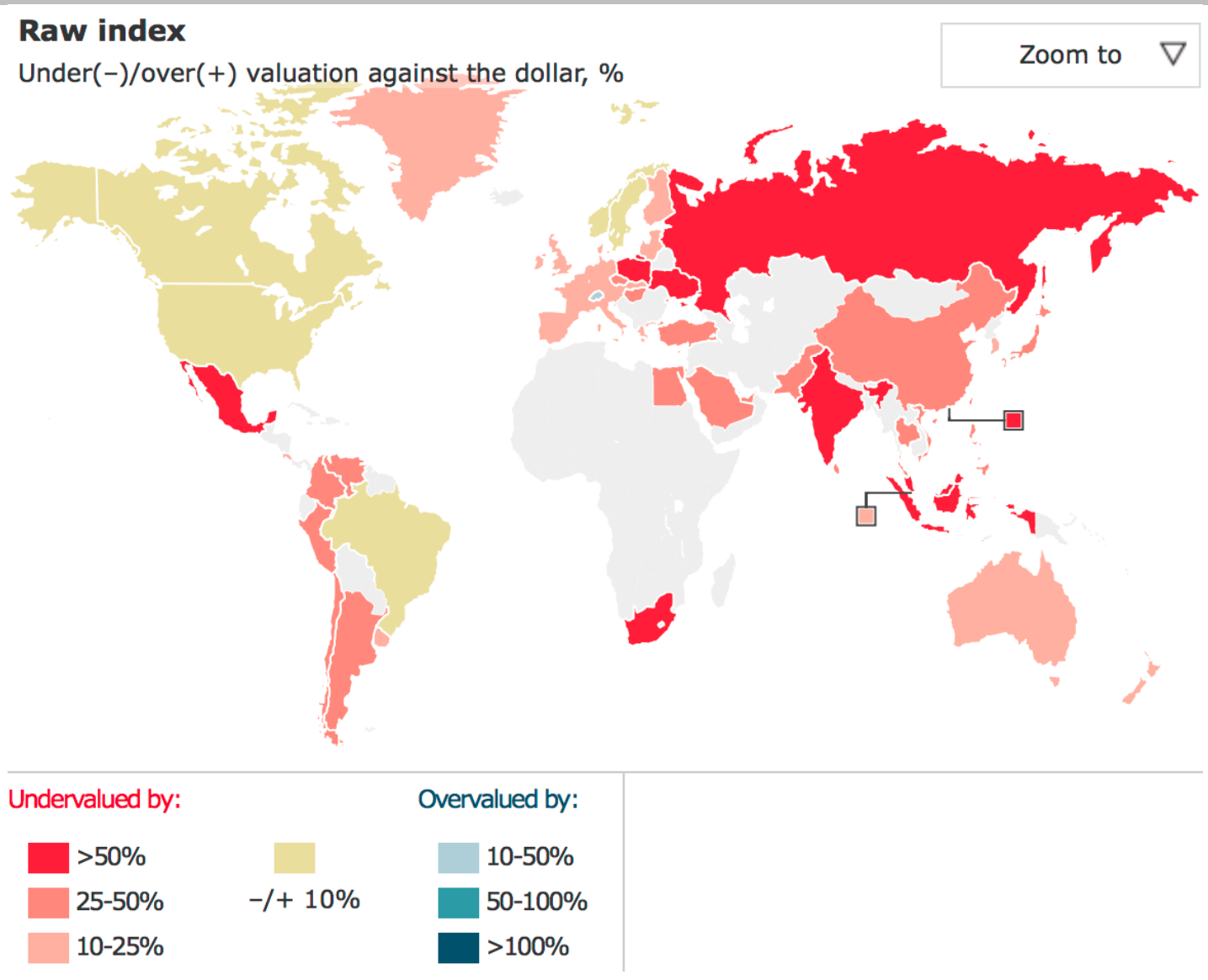
PPP exchange rate: R-dollar=2/3 P dollar $\longrightarrow$ P dollar worth more.

Relative GDP using exchange rates: 120/20=6 (1-1 ex. rate).

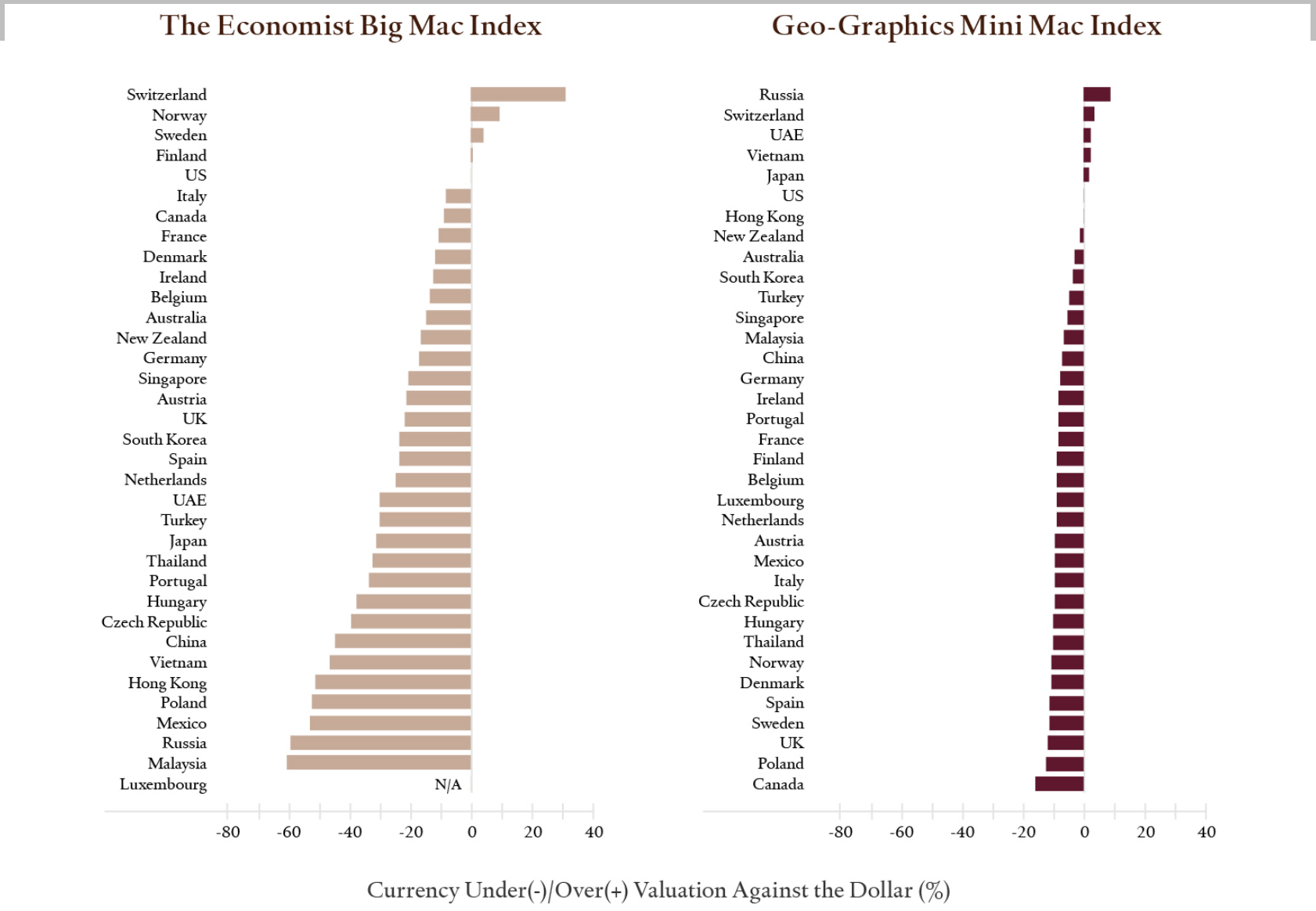
Relative GDP using PPP exchange rates:

$$\frac{120R}{20P} = \frac{120}{20}\frac{2}{3} = 6\frac{2}{3} = 4.$$

Comparing «Output» across countries: The Big Mac Index



Comparing «Output» across countries: Big Mac - Ipad mini



Sources: Apple, Bloomberg, The Economist
Credits: Benn Steil, Emma Smith, Julia Ro

COUNCIL on FOREIGN RELATIONS

Measuring «Output»: per-capita GDP

- Population grows over time => if output grows at the same rate as population, income per capita remains the same
- Hence, we calculate output per capita:
  => nominal or real GDP/population (i.e., y=Y/N)
- This also helps to compare income across countries
- Output per capita measures economic progress better (Adam Smith, *The wealth of Nations...*)

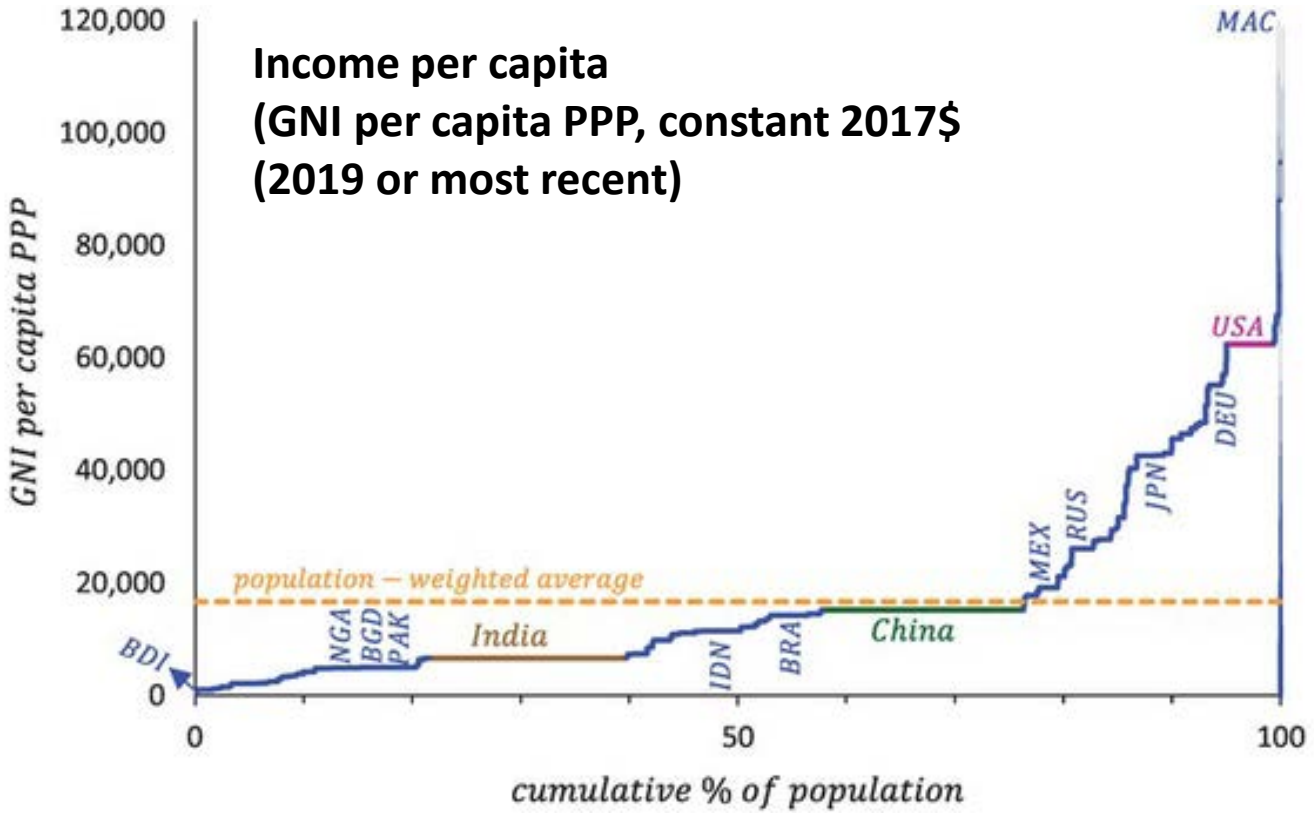
Top Eleven Countries according to Three Different Measures (2009)

Rank	Highest GDP per Capita		Largest Economies		Most Populous Countries	
	Country	GDP per Capita ($)	Country	Total GDP ($ trillions)	Country	Population (millions)
1	Qatar	159,469	United States	12.62	China	1,320
2	Luxembourg	84,525	China	10.08	India	1,160
3	United Arab Emirates	52,946	Japan	3.81	United States	307
4	Bermuda	52,090	India	3.76	Indonesia	240
5	Macao	51,057	Germany	2.66	Brazil	199
6	Norway	49,945	United Kingdom	2.07	Pakistan	181
7	Singapore	47,373	Russia	2.05	Bangladesh	154
8	Kuwait	46,639	France	1.98	Nigeria	149
9	Brunei	46,229	Italy	1.68	Russia	140
10	Australia	41,304	Brazil	1.62	Japan	127
11	United States	41,099	Mexico	1.29	Mexico	111

You can see some correlation between Total GDP and Population

... but both are quite uncorrelated with GDP per Capita!

Huge differences across countries

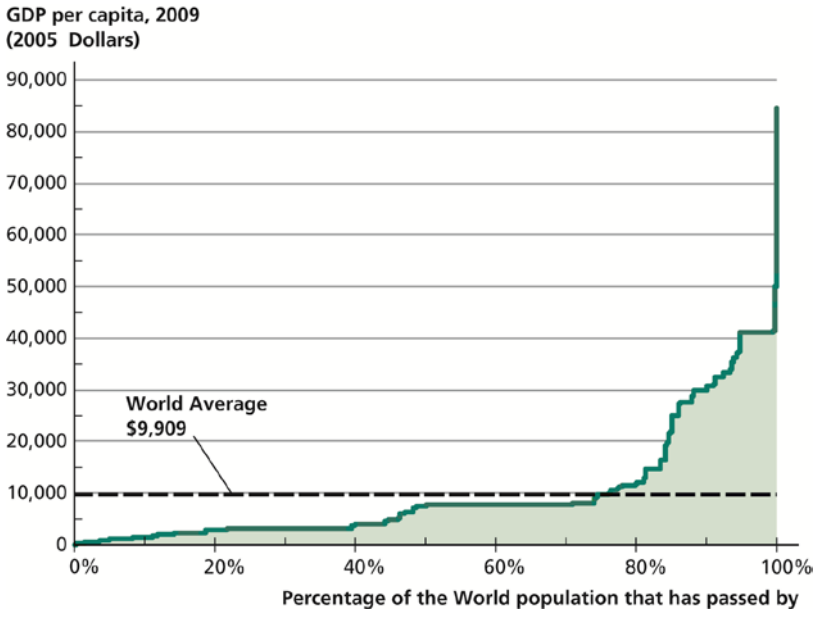


Notes: 191 countries included; figure based on country averages.
Source: World Development Online data.

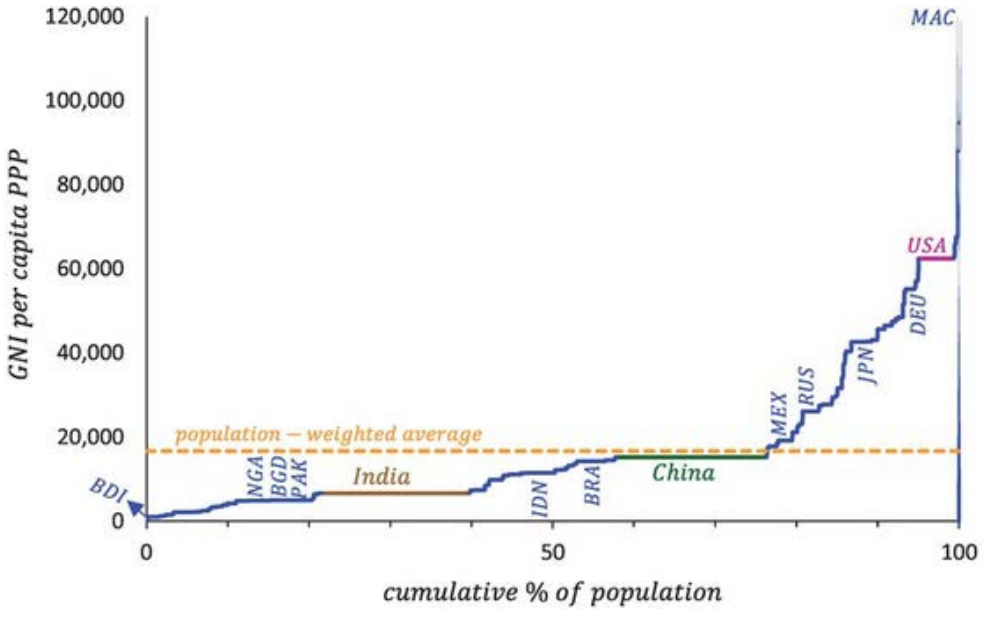
- Only the top 20% of world population has income > average
- Top 10 % of world population has income > $40,000 (Top 5% > $60,000)

Huge differences across countries

Income per capita:
GDP per capita, constant 2005 $ (2009)

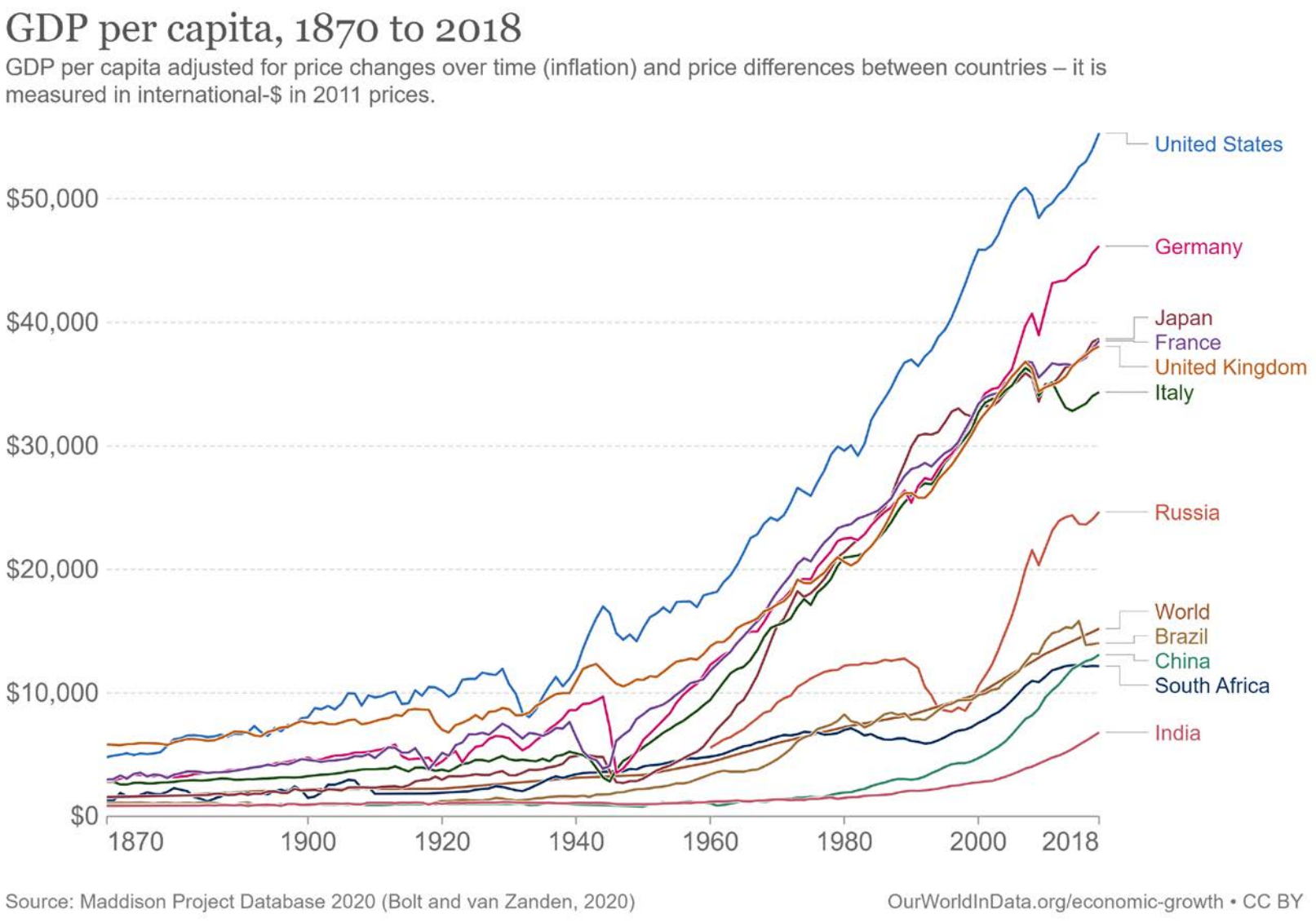


Income per capita:
GNI per capita (PPP), constant 2017 $ (2019 or most recent)



…quite similar pictures!!!

GDP per Capita (1870–2018)



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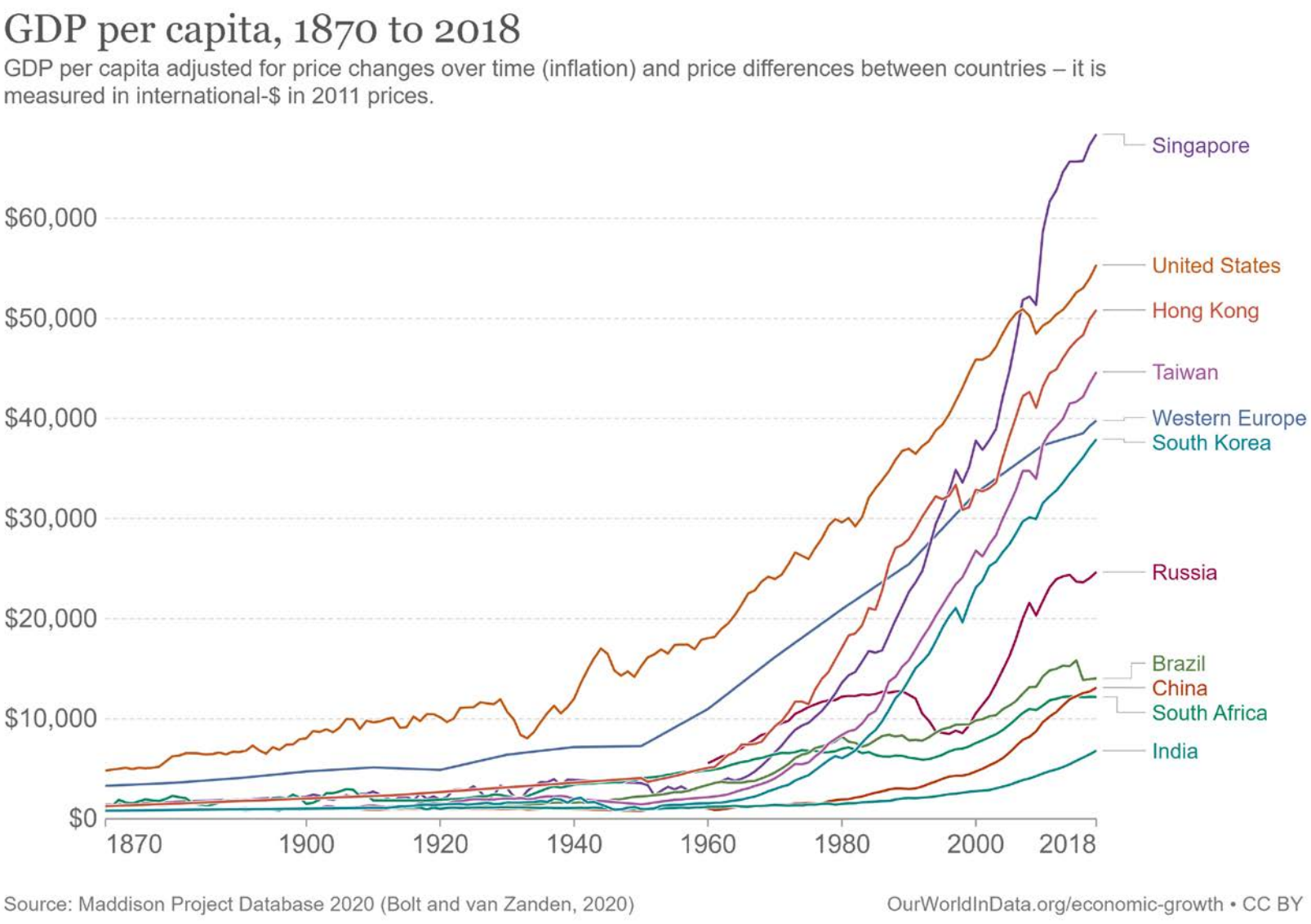


Measuring «Output»: per-worker GDP (L productivity)

- Output grows as a result of increase in the number of workers, but also as a result of the ability of workers to produce more.

- To capture this we calculate output per worker, which is also called labor productivity: : realGDP/n.workers (i.e., y=Y/L).

- Output per worker can also be calculated as output per hour, depending on the quality of data.

Measuring «Output»: GDP growth rate

Discrete time calculation of the rate of growth

- The rate of growth is measured as the rate in which output grows every year. In period t the rate of growth is:

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

- The rate of growth in Italy in 2015 (units of output are in Euros of 2010):

$$\frac{1{,}553.88 - 1{,}542.59}{1{,}542.59} = 0.00732$$

- To express the rate of growth in percentage value, multiply by 100: the rate of growth in Italy in 2012 was 0.732 %.

Measuring «Output»: GDP growth rate

Continuous time calculation of the rate of growth (ratio scale)

- Sometimes we use a continuous calculation of the rate of growth:

$$\ln Y_t - \ln Y_{t-1}$$

- According to this calculation, the rate of growth in Italy in 2015 was:

$$\ln(1{,}553.88) - \ln(1{,}542.59) = 0.00729$$

- Hence, the two calculations yield very similar results.

Measuring «Output»: GDP growth rate

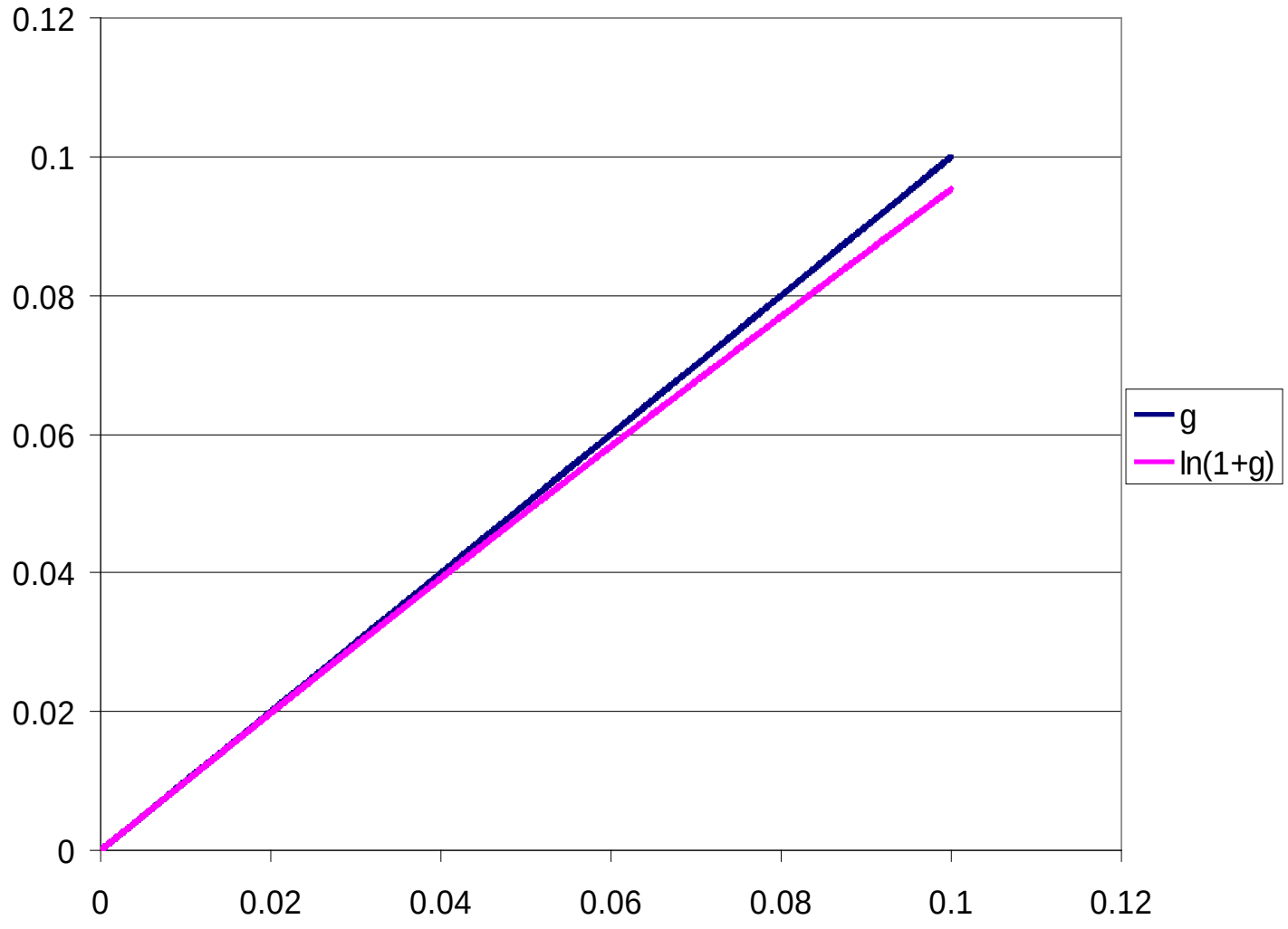
- The second calculation is based on the right calculation if the time is continuous.

- Of course output is measured at most quarterly, and in economic growth, that studies long-run developments, annual data is sufficient, but for low rates the two calculations are very close.

- The reason is: $\ln Y_t - \ln Y_{t-1} = \ln \frac{Y_t}{Y_{t-1}} = \ln\left(1 + \frac{Y_t - Y_{t-1}}{Y_{t-1}}\right) \cong \frac{Y_t - Y_{t-1}}{Y_{t-1}}$

- And this is a good enough approximation as shown by the graph ➡

Measuring «Output»: GDP growth rate



Measuring «Output»: GDP growth rate

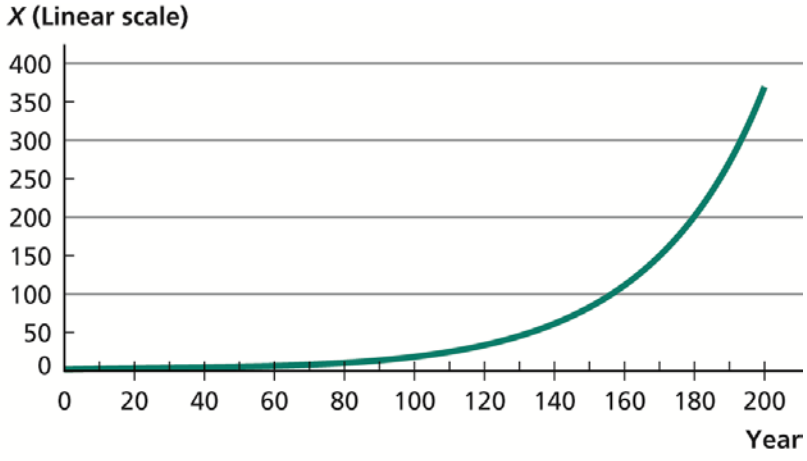
- The continuous rate of growth yields a very helpful way to present data on output in graphs.

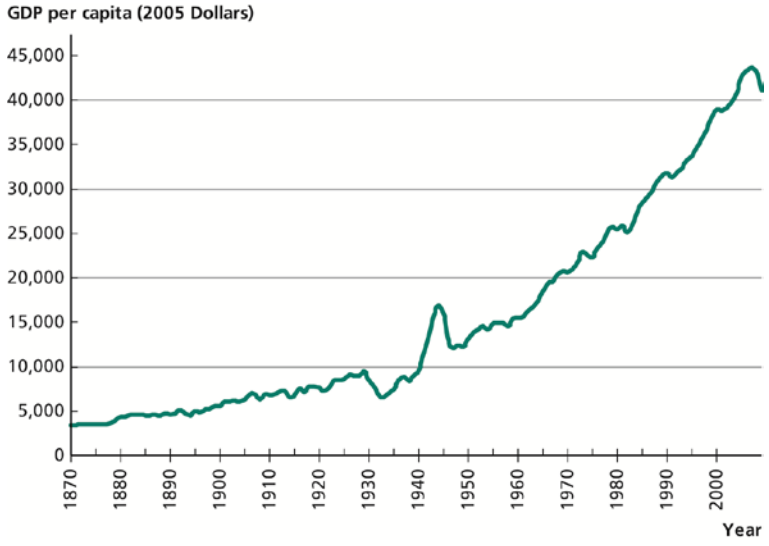
- If we plot ln(Y) against T (years)
  → the difference on the Y axis is growth rate
  → the slope of the graph is annual rate of growth.

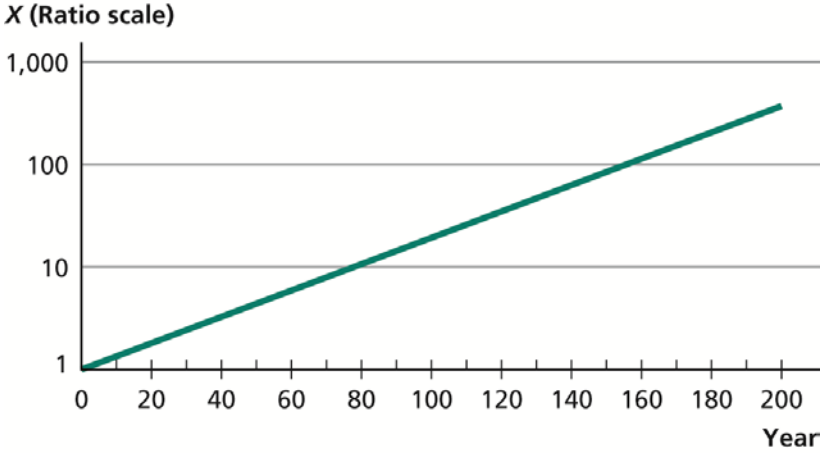
- This even enables us to see from the graph the average rate of growth in a period, as it is the slope of the line from the beginning to the end:

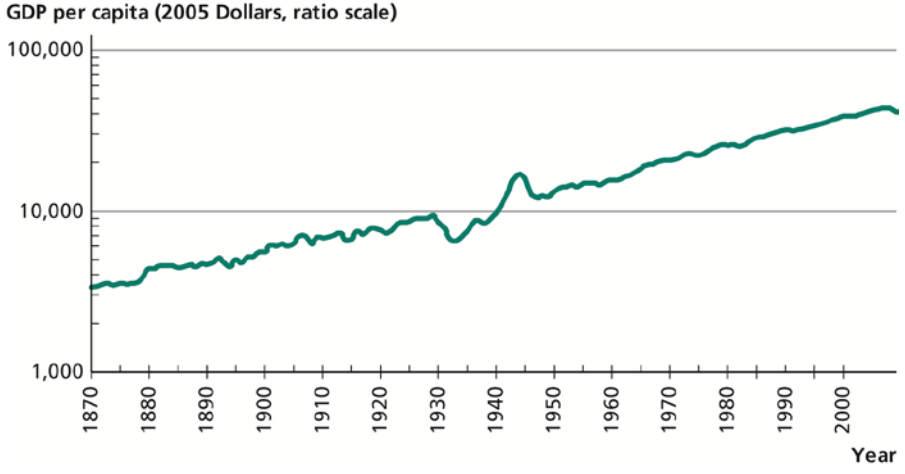
- Average rate of growth from 0 to T is: $\frac{\ln Y_T - \ln Y_0}{T}$

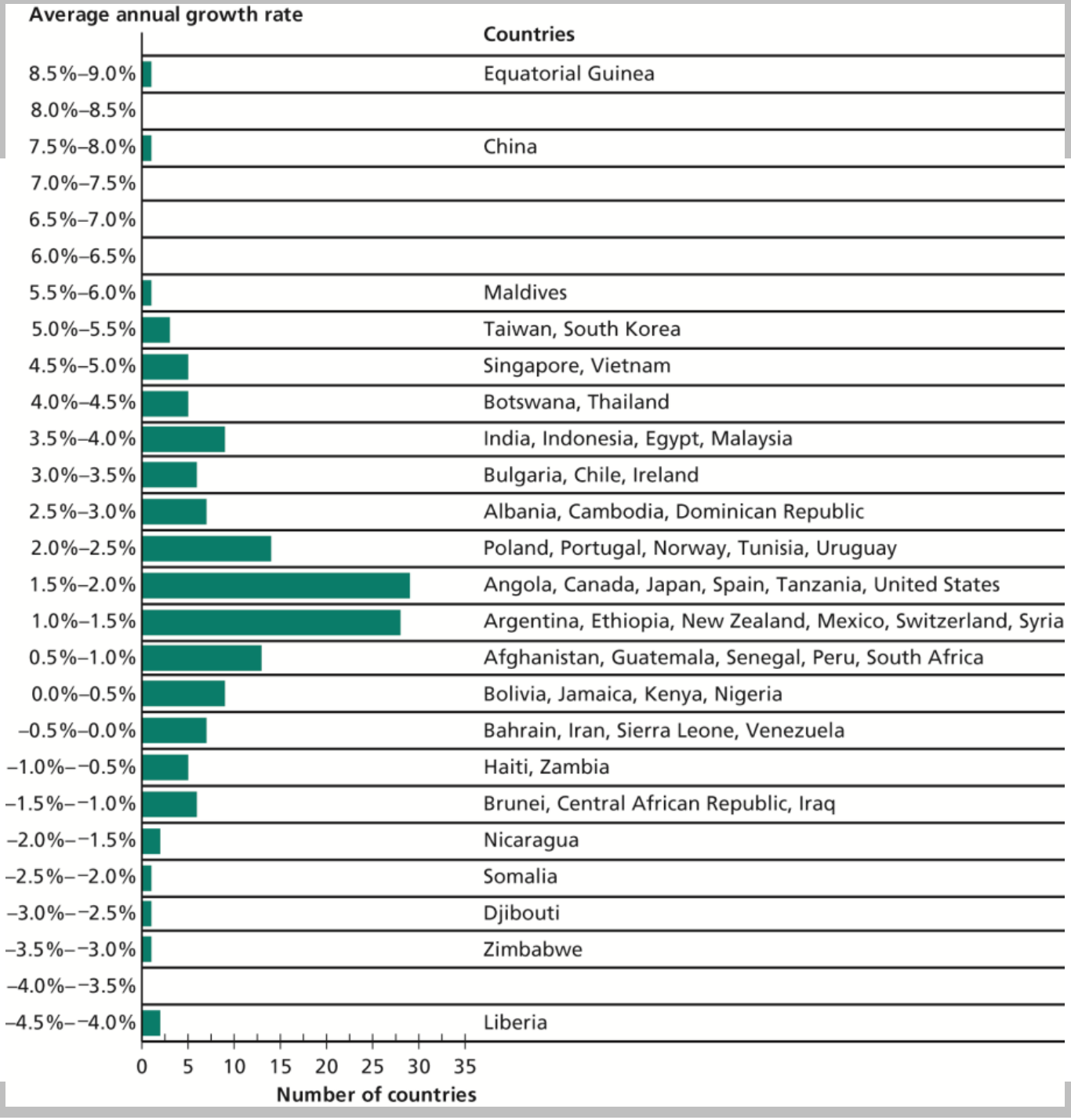
GDP growth rate: the effect of using a “ratio scale”





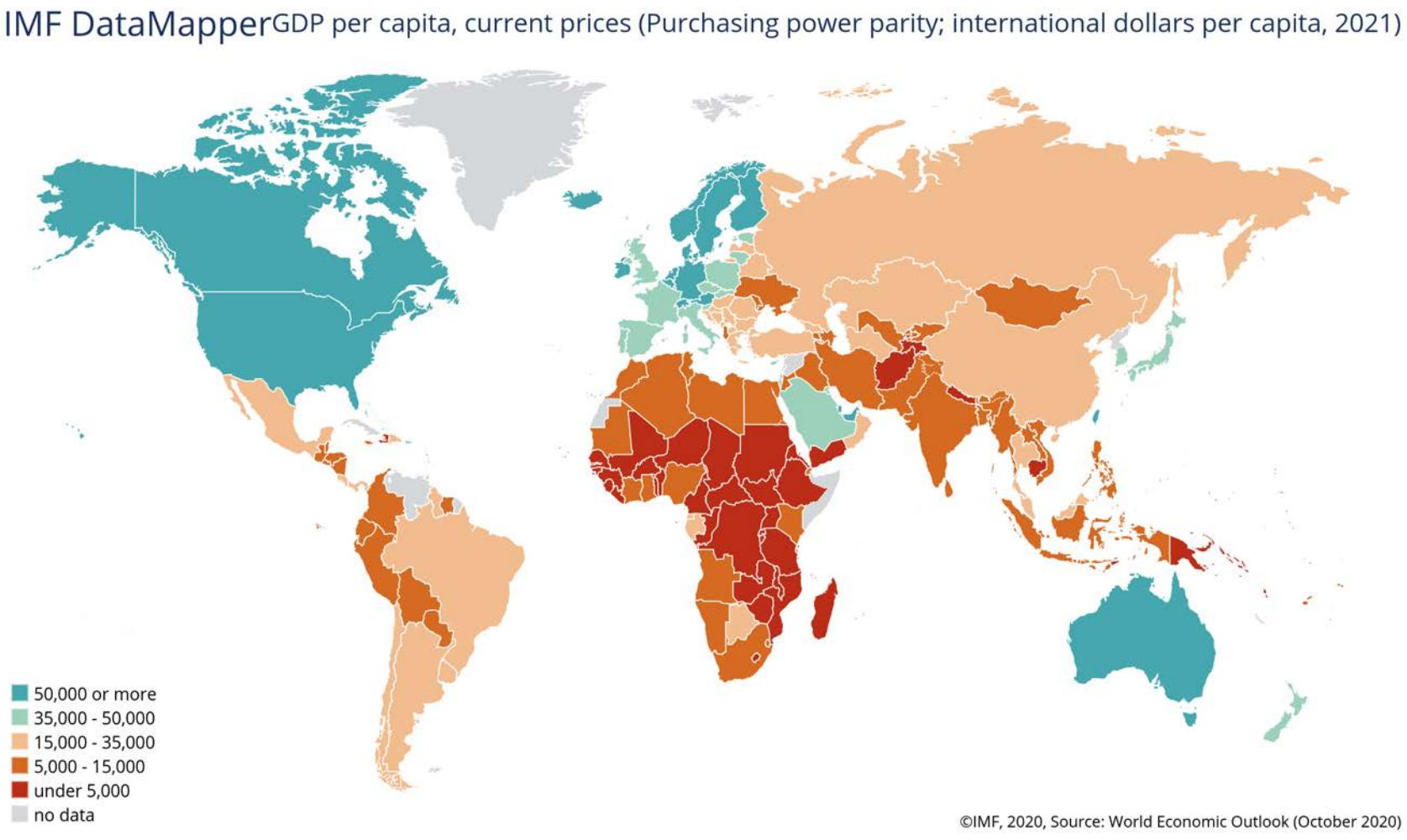






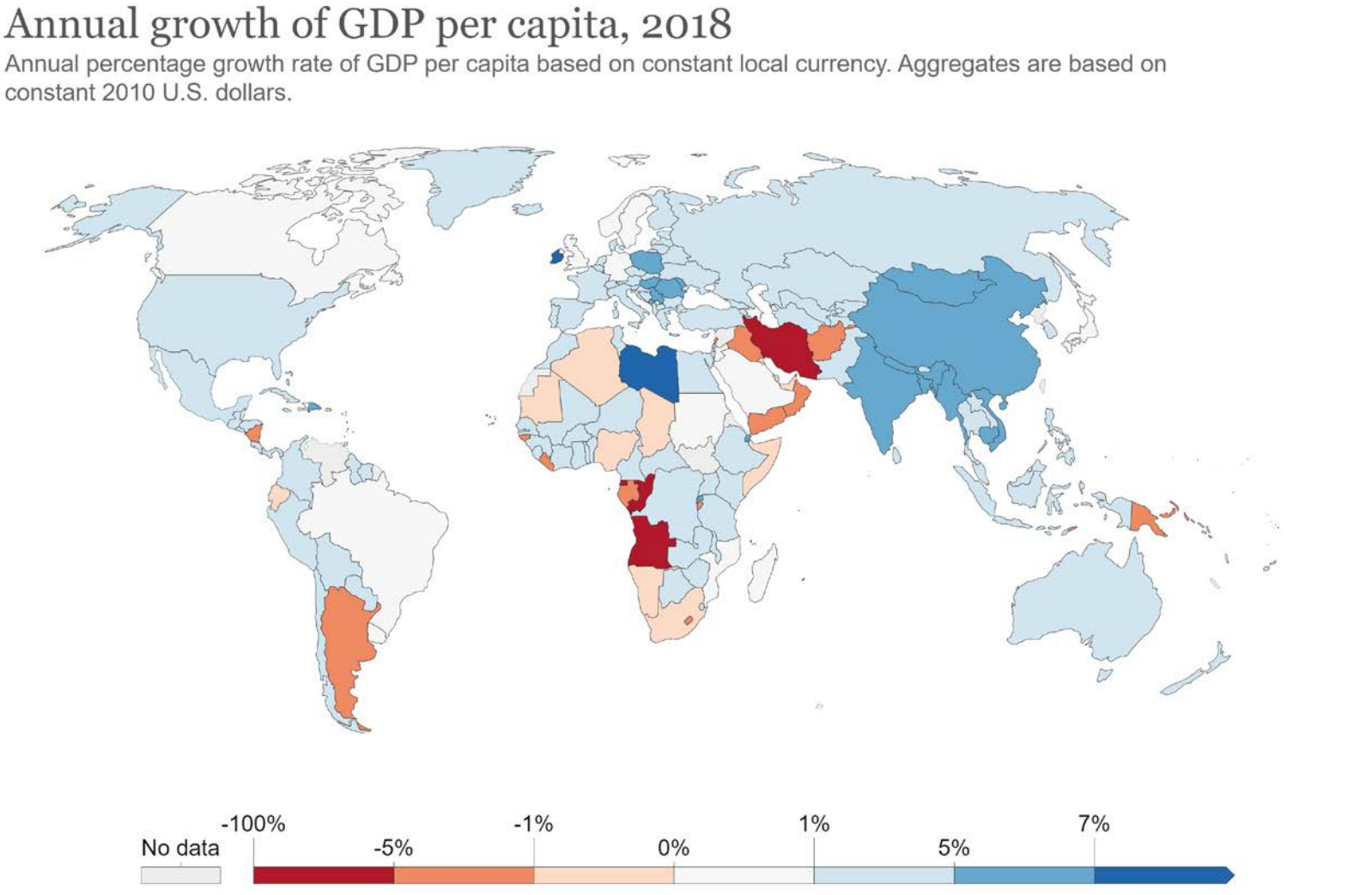
The Distribution of Growth Rates (1975–2009)

GDP per capita (PPP, 2020)

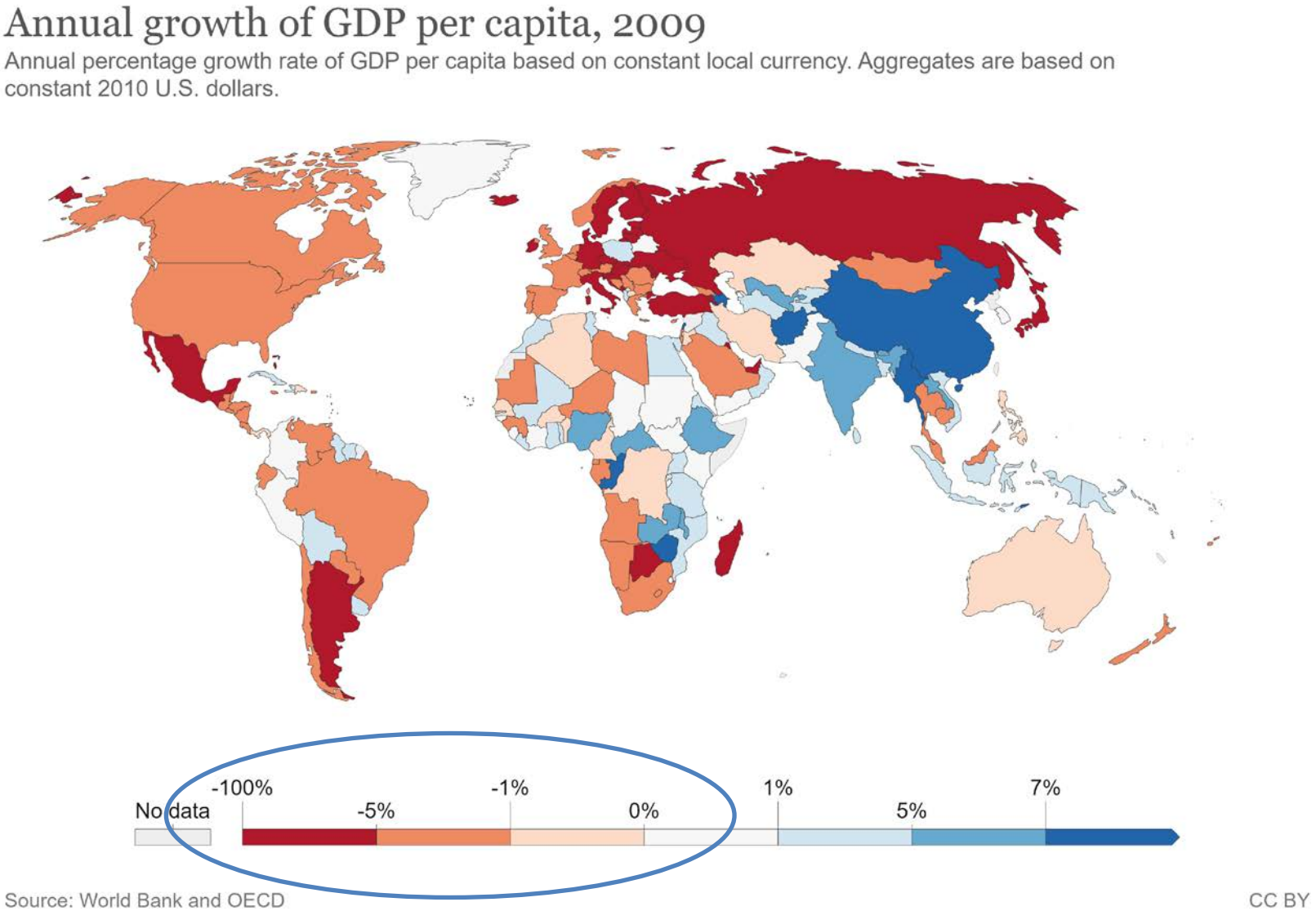


Long run: https://ourworldindata.org/grapher/maddison-data-gdp-per-capita-in-2011us?time=2018

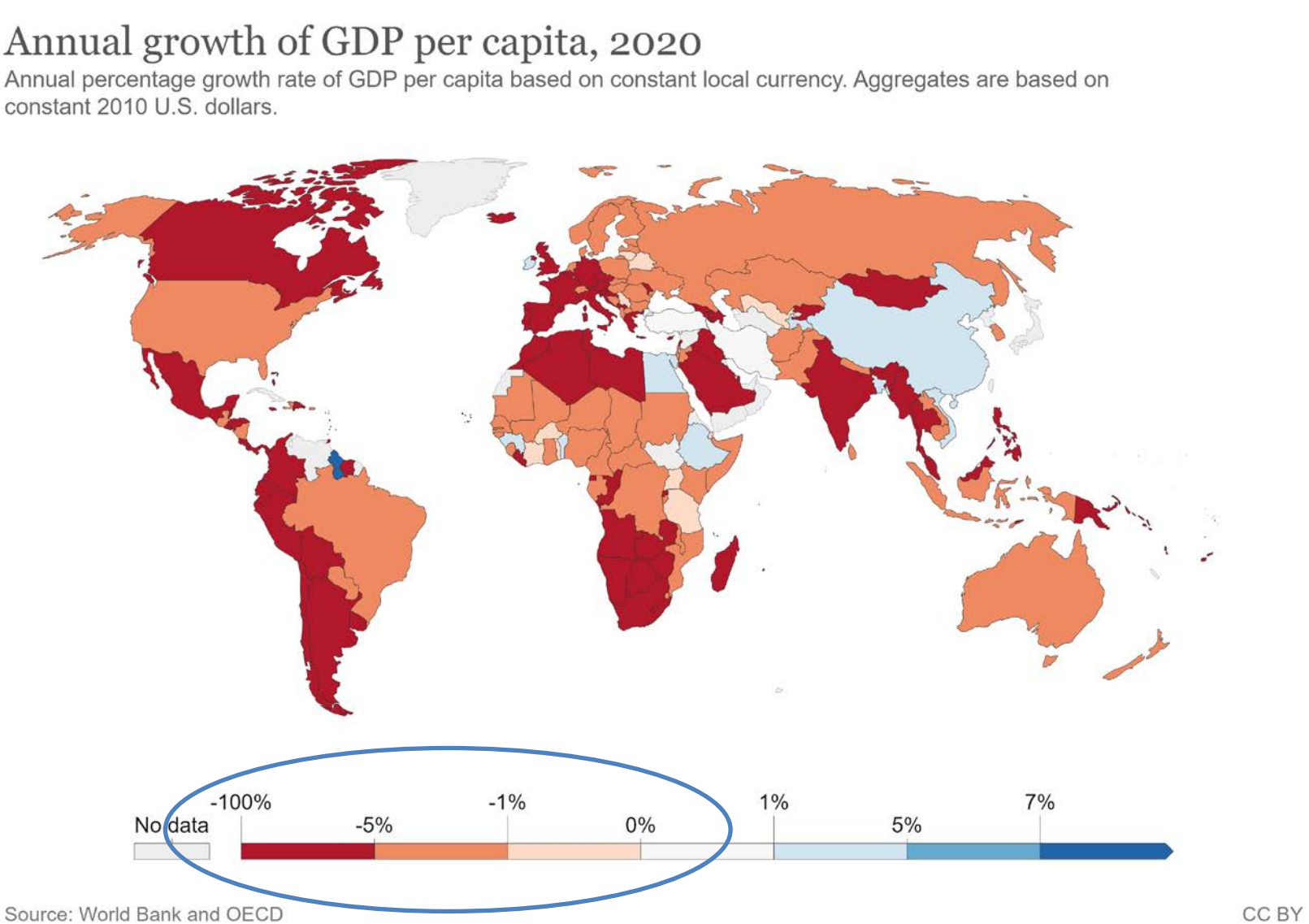
GDP growth rate (2018)



GDP growth rate (2009) – Economic Crisis



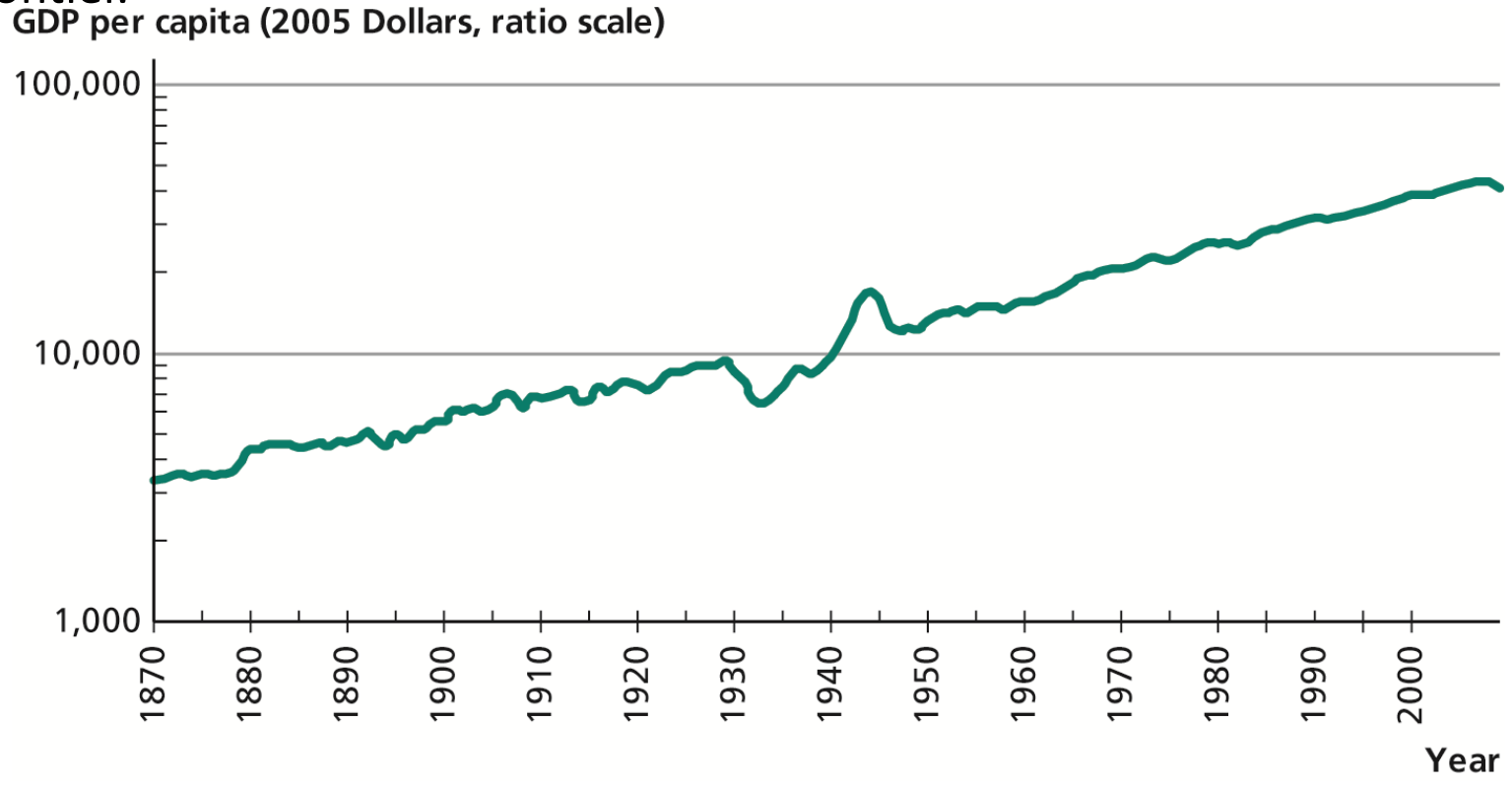
GDP growth rate (2020) – COVID Crisis



GDP growth rates of developed countries (long run)

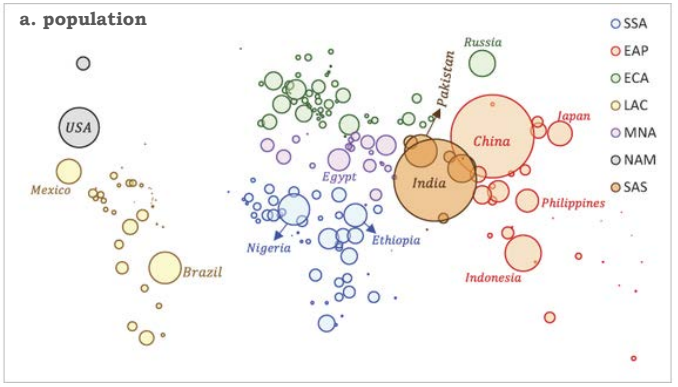
Long run perspective (trend): the trend of growth of the US has been quite stable over time, despite a big slump in the Great Depression and a rise in WW II

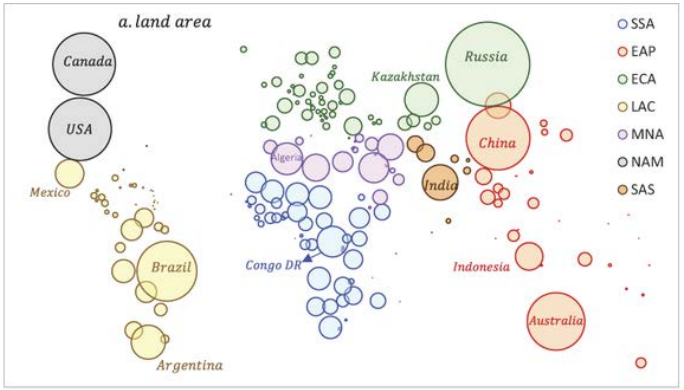
=> This is why the US is viewed as the leader in global growth, as the global frontier.

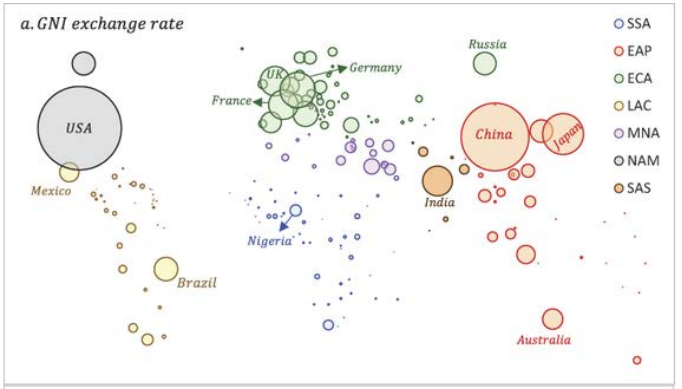


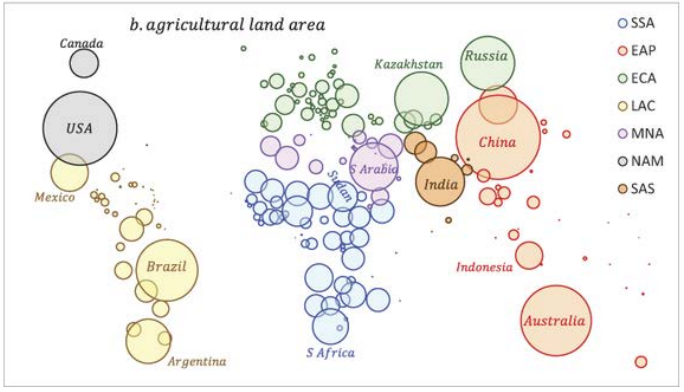
Real GDP growth rates

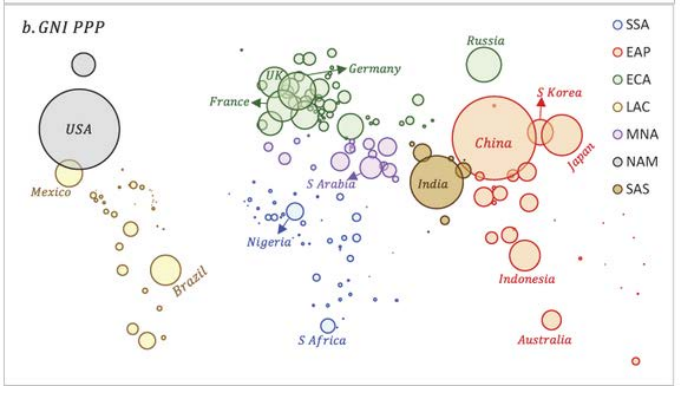
How different the World looks like…



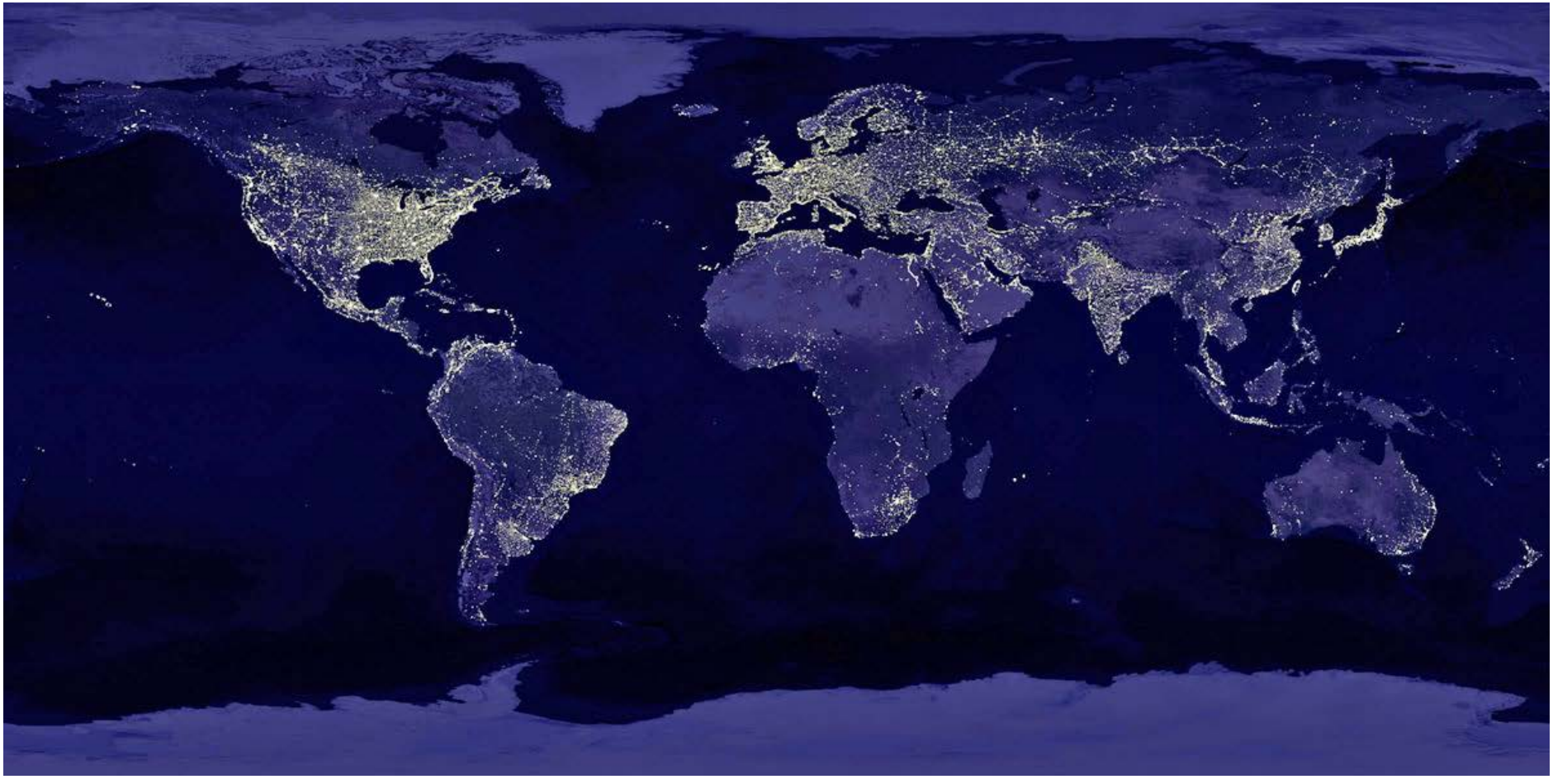




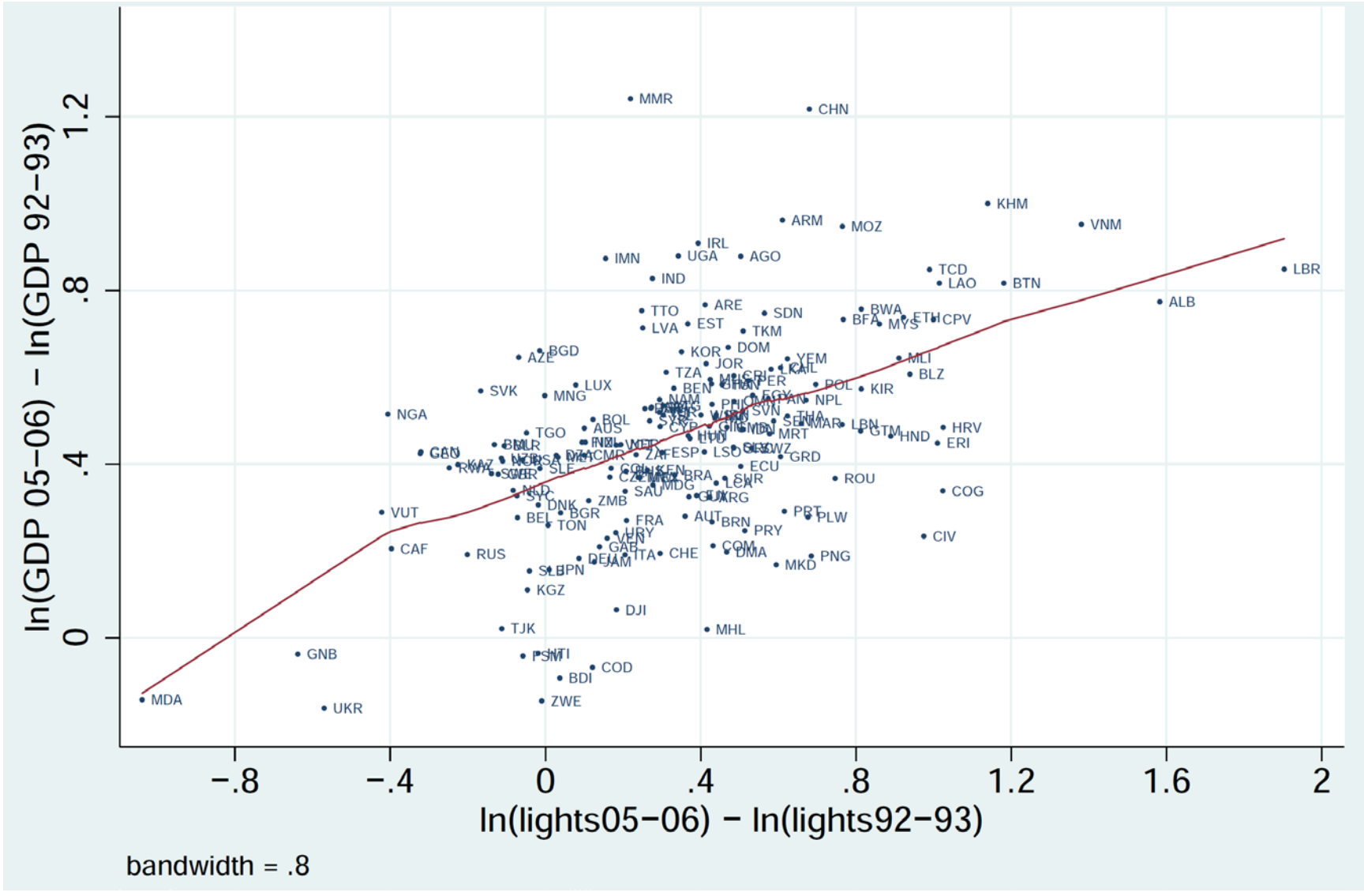




Night time lights: World



Growth in lights correlated with growth in GDP



(Henderson, Storeygard, Weil (AER, 2012)

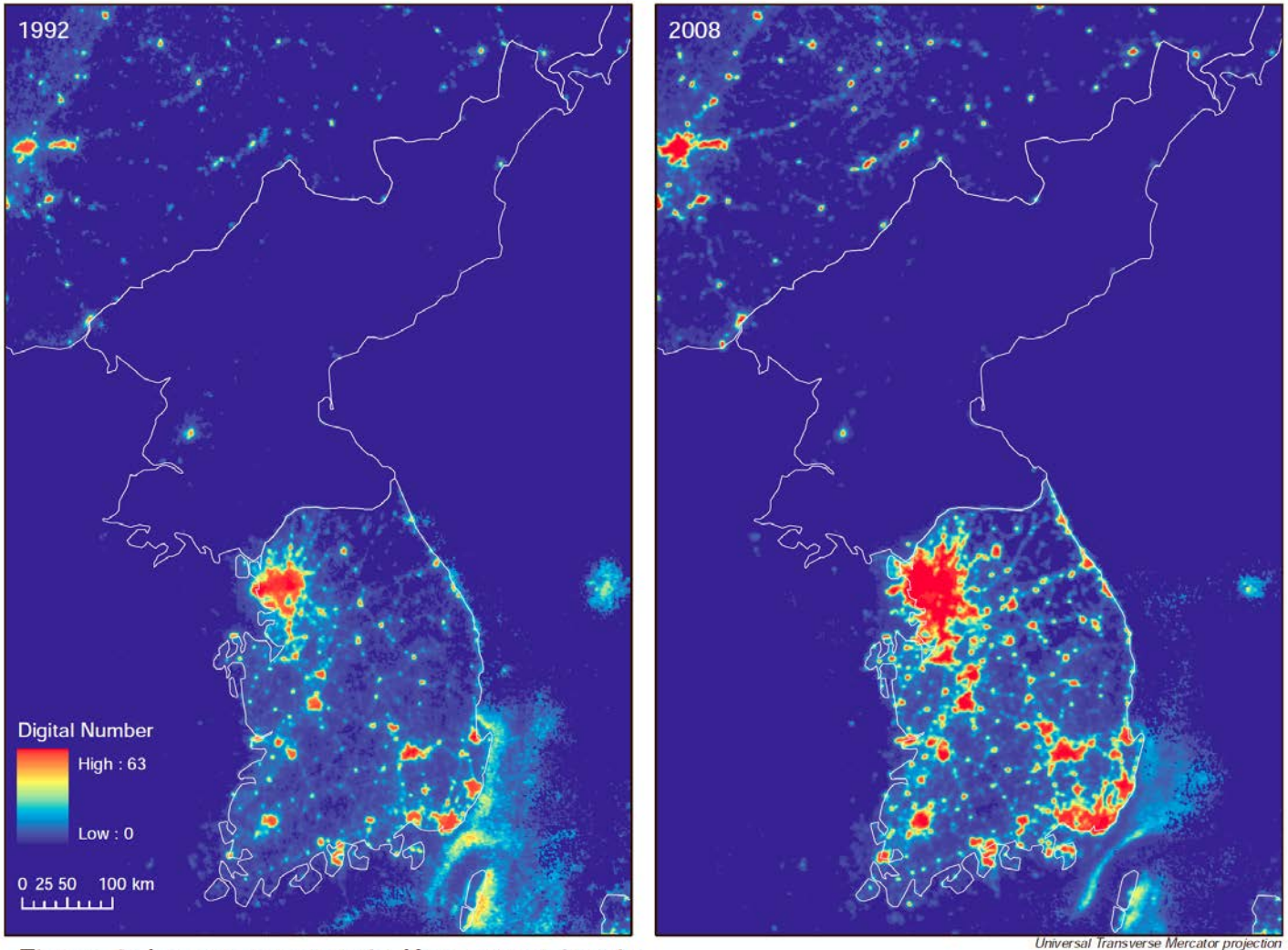
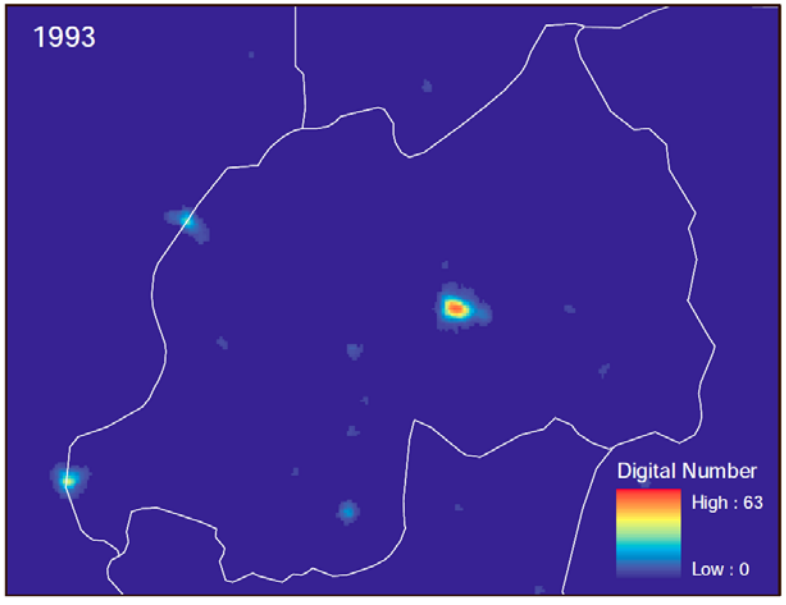
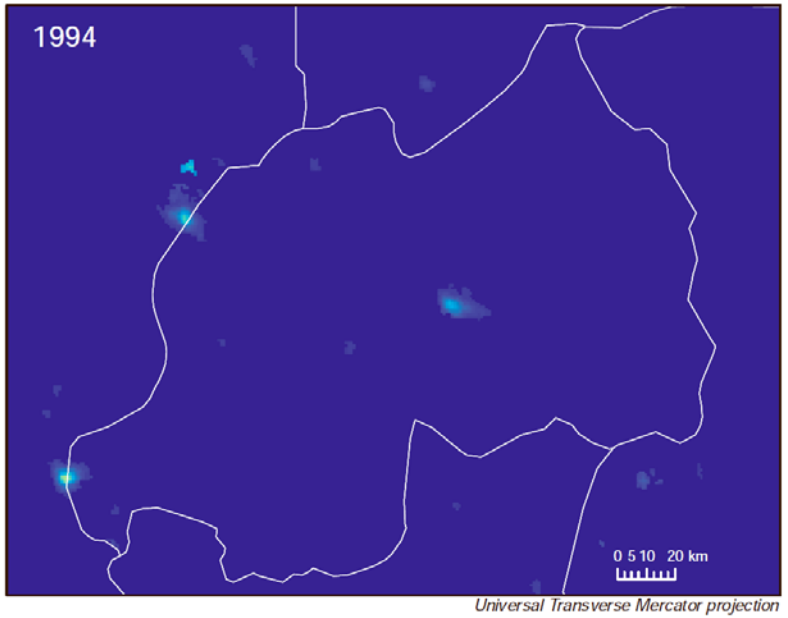


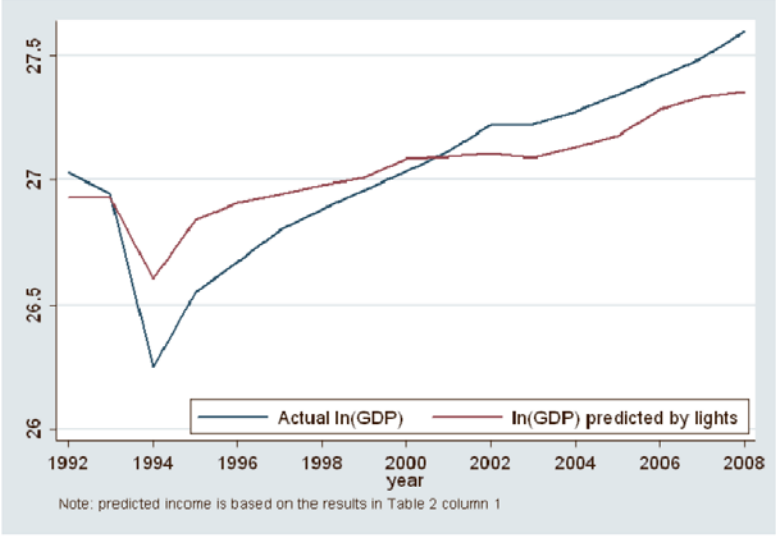
Figure 2: Long term growth: Korean peninsula

Real GDP increase 119%

(Henderson, Storeygard, Weil (AER, 2012)







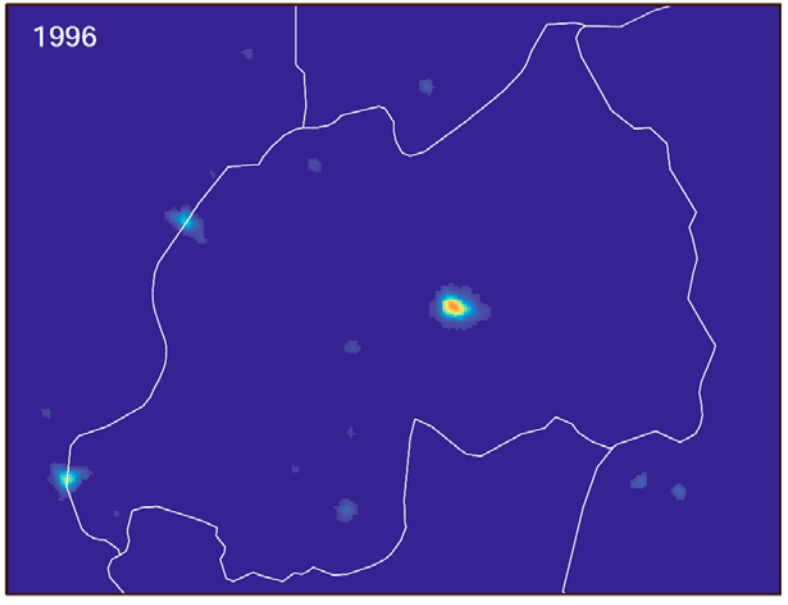


Figure 4: Genocide event: Rwanda

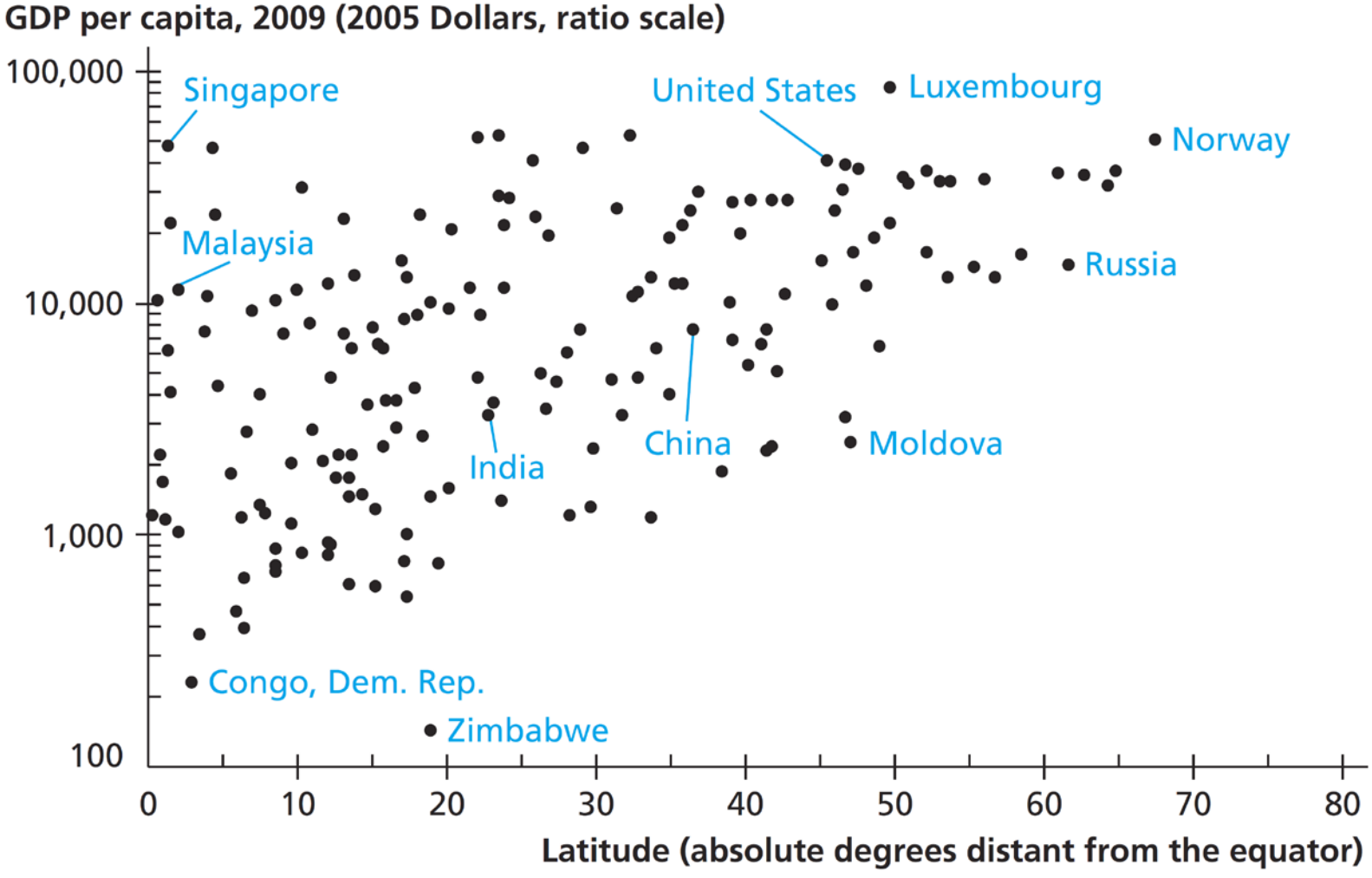
Night time lights: Africa

Night time lights: Asia

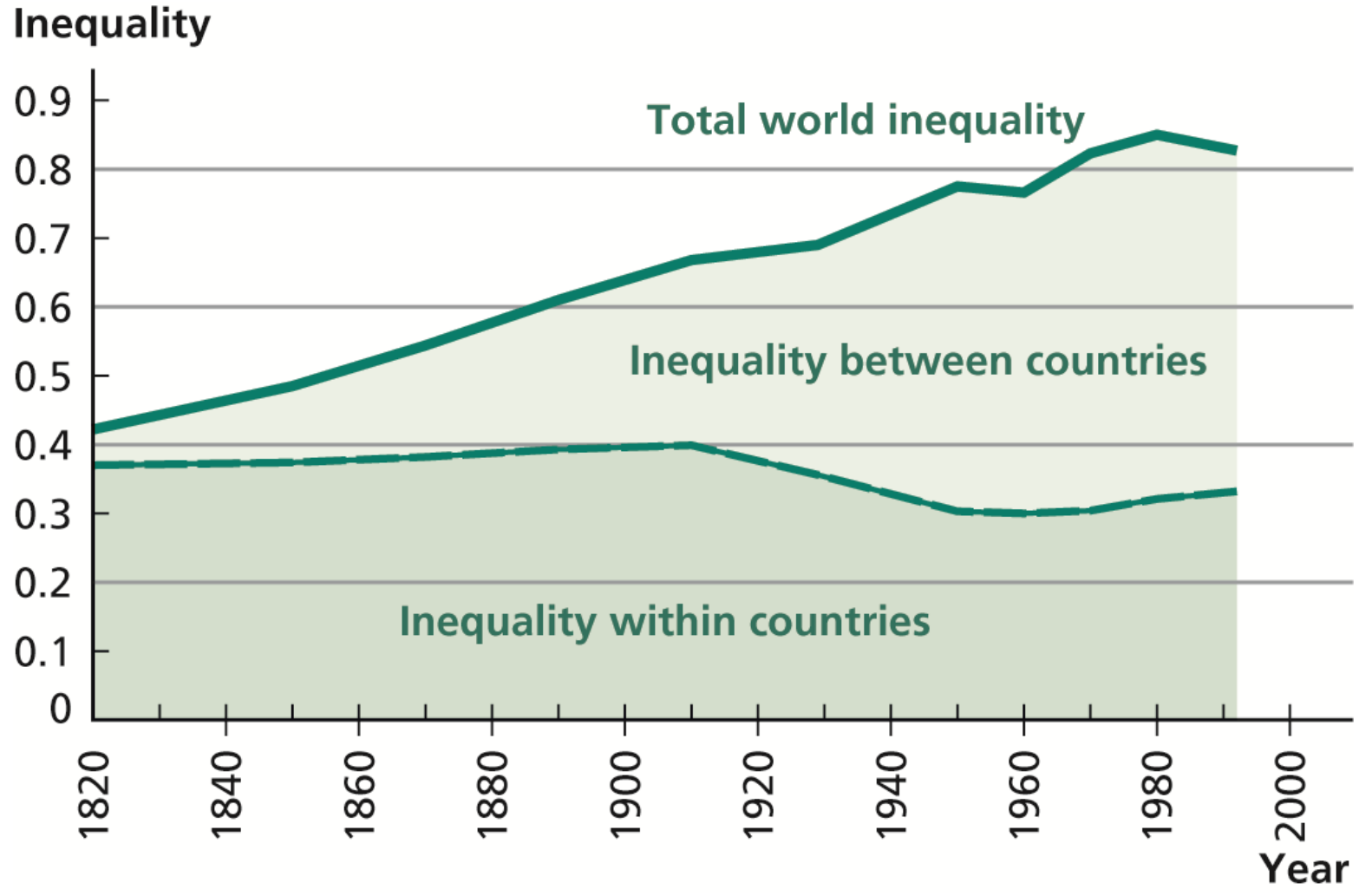
Night time lights: Europe

Night time lights: USA

Latitude and Income per Capita



World Inequality and Its Components, 1820–1992



Source: Bourguignon and Morrison (2002).

Real GDP growth rates

- GDP fluctuations around the trend represent business cycles.

- It is important not to confuse the two. A high growth rate in a year may be a result of coming out of a recession and say nothing about long-run economic growth. For that we need to look at the trend (i.e., the average)

  => Economic growth is about the “trend”

Real GDP growth rates

Take home facts:

- Gaps between countries are large and increasing
- Gaps between countries increased mostly in the last two hundred years
- Gaps within countries slightly decreased in the last century
- Economic growth follows geographical patterns
- Gaps are driven by the fact that some countries do not succeed in catching up with the leading countries in economic growth

Growth rates: tips and tricks

- If x grows at rate g, then

$$x_{t+1} = x_t(1+g)$$

- If the growth rate is g in two years, then

$$\begin{aligned} x_{t+2} &= x_{t+1}(1+g) \\ &= x_t(1+g)(1+g) \\ &= x_t(1+g)^2 \end{aligned}$$

- If growth is g in n years, then

$$x_{t+n} = x_t(1+g)^n$$

Growth rates: tips and tricks

- Average growth rate:

$$y_{t+n} = y_t \cdot (1+g)^n$$

- In logs:

$$\ln(y_{t+n}) = \ln(y_t) + n\ln(1+g)$$

- ln is the natural log: $\ln(e^x) = x$.
- For small growth rates:

$$\ln(1+g) \approx g \quad \Rightarrow \quad \ln(y_{t+n}) = \ln(y_t) + ng$$

(check this by example!)

- Therefore:

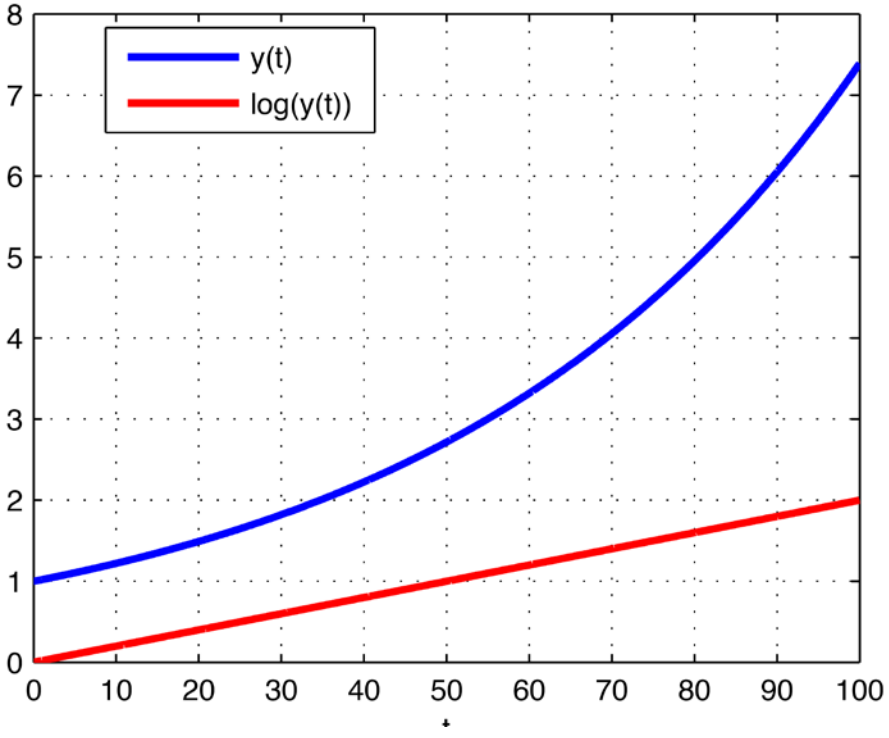
Average Growth rate $g = \dfrac{\ln(y_{t+n}) - \ln(y_t)}{n}$ ⇒ growth rate

⟹ n. of years

Growth rates: tips and tricks

This economy grows at a constant rate. However, the blue graph (linear scale) looks like if growth were accelerating.

→ To show that something grows at a constant rate, just plot its log (ratio scale - red graph)



Are you able to say
- at what rate did this economy grow between year 50 and year 100?
- What is the avg growth rate?

$$\ln(y_{t+n}) = \ln(y_t) + ng$$

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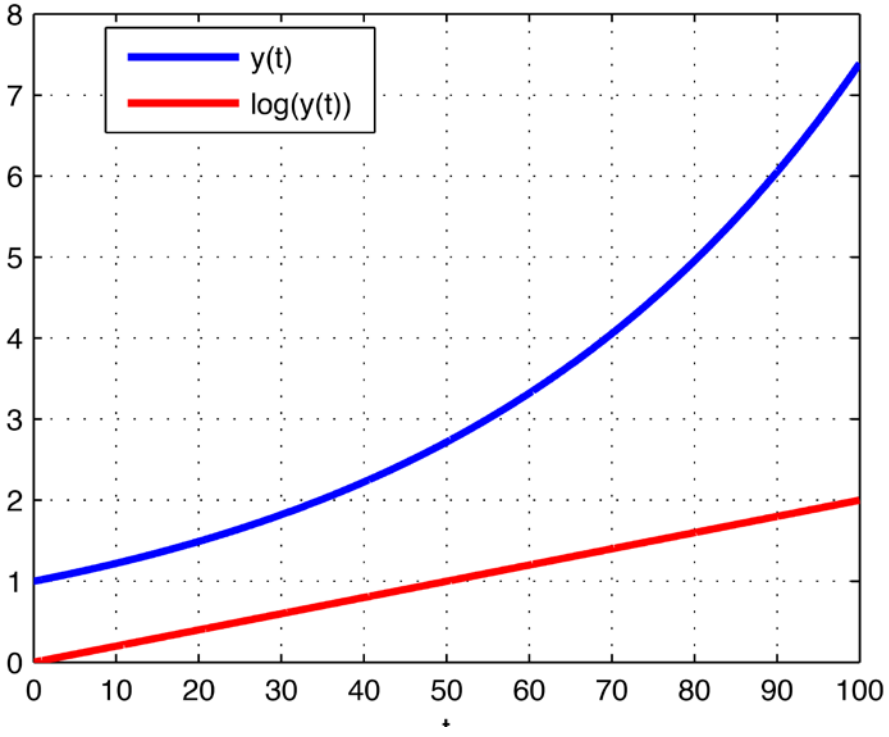

Are you able to say

- at what rate did this economy grow between year 50 and year 100?

YES: 2-1=100%

- What is the avg growth rate?

100%/50=2%

$$\ln(y_{t+n}) = \ln(y_t) + ng$$

Growth rates: tips and tricks

- Assume we know x_{t+n} and x_t. What is the average annual growth rate?
- We know

$$x_{t+n} = x_t (1+g)^n$$

- Rearranging,

$$g = \left(\frac{x_{t+n}}{x_t} \right)^{1/n} - 1$$

- Example: Let $x_t = 100$ and $x_{t+10} = 150$. Then

$$g = \left(\frac{150}{100} \right)^{1/10} - 1 = 0.041.$$

Growth rates: tips and tricks

- China's GDP/capita is 1/6 of Norway's GDP/capita.
- Growth in China is 10% and growth in Norway is 2%.
- When will China's GDP/capita surpass Norway's?

$$
\begin{aligned}
x_{Norway}\,(1+0.02)^n &= x_{China}\,(1+0.10)^n \\
1.02^n &= \frac{1}{6}1.10^n \\
n\ln 1.02 &= \ln\frac{1}{6} + n\ln 1.10 \\
n &= \frac{\ln(1/6)}{\ln 1.02 - \ln 1.10} \\
&\approx 24
\end{aligned}
$$

Growth rates: tips and tricks

- Often convenient to write down models in continuous time.
- Can be shown that (partial differential equation)

$$y(t) = y(0)\, e^{gt}$$

  where $y(0)$ is the initial value, g is the growth rate and t is time.
- Using the properties of logarithms, we get

$$\begin{aligned} \ln y(t) &= \ln \left[y(0)\, e^{gt} \right] \\ &= \ln y(0) + \ln e^{gt} \\ &= \ln y(0) + gt \end{aligned}$$

$\longrightarrow$ The growth rate g is the slope coefficient of e.g. log GDP plotted against time.

- We denote $\dot{y} = \partial y / \partial t$ and $\hat{y} = \dot{y}/y$.
- Observe that $\dot{y} = y(0)\, g e^{gt}$ and $\hat{y} = g$.

Growth rates: tips and tricks

- How long does it take for income to double if growth is constant?
- We have

$$y(t) = y(0)\, e^{gt}$$

- And want to find $y(t) = 2y(0)$, or

$$\begin{aligned} y(0)\, e^{gt} &= 2y(0) \\ e^{gt} &= 2 \\ gt &= \ln 2 \\ t &= \frac{\ln 2}{g} \approx \frac{0.70}{g} = \frac{70}{\%\text{ annual growth}} \end{aligned}$$

- If $g = 0.02$, then $t = 35$.
- If $g = 0.08$, then $t \approx 9$.

Growth rates: tips and tricks

Define

$$\hat{x} = \frac{\dot{x}}{x}$$

We know

$$\frac{\partial \ln y}{\partial t} = \frac{\dot{y}}{y} = \hat{y}$$

Then we have

$$\widehat{xy} = \frac{\partial \ln (xy)}{\partial t} = \frac{\partial \ln x}{\partial t} + \frac{\partial \ln y}{\partial t} = \hat{x} + \hat{y}$$

$$\widehat{x/y} = .. = \hat{x} - \hat{y}$$

$$\widehat{x^a} = .. = a\hat{x}$$

E.g. if x and y grow by 2% and 4%, then xy grows by 6%.